MONTHLY NOTICES

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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Captain Richard Algernon Craigie Daunt, D.S.O., Lynalta, Newtownards, Co. Down, Ireland,
and
Edgar Odell Lovett, Ph.D., Professor of Astronomy, Princeton University, New Jersey, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Warin Foster Bushell, The Hermitage, Harrow (proposed by Col. G. L. Tupman);
Charles P. Butler, A.R.C.Sc., F.R.P.S., Solar Physics Observatory, South Kensington, S.W. (proposed by William J. S. Lockyer);
William Doberck, Ph.D., late Director, Hong Kong Observatory, Kowloon, Elgin Road, Sutton, Surrey (proposed by Sir W. H. M. Christie);
James Nangle, Technical College, Sydney, N.S. Wales, Australia, and Private Observatory, Tupper Street, Stanmore, near Sydney (proposed by C. J. Merfield);
Charles W. Raffety, Wynnstay, Woodcote Valley Road, Purley, Surrey (proposed by Richard Kerr); and
Rev. T. J. Williams-Fisher, M.A., Rector of Norton, Atherstone (proposed by Rev. D. H. Sparling).

Seventy-one presents were announced as having been received since the last meeting, including amongst others :—Lieut. A. ff.

Garrett, The Jaipur Observatory and its builder, presented by the author; Greenwich Astrographic Catalogue, vol. ii., 16 charts of the Astrographic Chart of the heavens, and Observations of the planet Eros, 1900–1901, presented by the Royal Observatory, Greenwich; Professor E. S. Holden, Galileo, and other tracts, presented by the author; Oxford Astrographic Catalogue, vol. iv., presented by the University Observatory, Oxford; Pennsylvania University Publications, Catalogue of 648 double stars discovered by Professor Hough, presented by the University; three lantern slides of the Corona of 1908 January 3, taken by Professor Campbell, presented by the Lick Observatory.

An Empirical Law of Astronomical Refraction.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following investigation was originally undertaken with the view of substituting a simple proof of the law of refraction for students who could not follow the more elaborate proof involving the differential equation. But it was a surprise to find how closely the observed refraction could be represented with so rough a supposition as that of three, or even two, homogeneous shells of atmosphere; and the question was suggested whether, in the present state of our knowledge, more elaborate hypotheses were really justified. If a rough supposition fits the facts, clearly it is no proof of the correctness of a more elaborate one that it also fits the facts.

2. Moreover, suspicion of the correctness of existing hypotheses was suggested from another direction. Meteorologists are finding that the temperature of the atmosphere does not follow a smooth gradient, as is generally assumed in refraction hypotheses: at a certain height a wholly unexpected state of things has been found to exist. According to M. Teisserenc de Bort there is above 10 or 12 km., an "isothermal layer" in which the temperature ceases to fall as we ascend; and the conditions are similar over parts of the world where the temperatures close to the surface differ widely. It is difficult to reconcile these results of observation with the hypotheses usually adopted in constructing tables of refraction.* It is, however, not intended to examine at present the consequences of taking M. Teisserenc de Bort's work into account;—merely to show that it may not be difficult to do so when we have fuller information, without, perhaps, dislocating existing refraction tables.

3. First let us consider an atmosphere of one homogeneous spherical layer. Let C be the Earth's centre; OM a section of its surface through C and the star; LB the section of the boundary of

* I am indebted to Mr. Saunder for a reference to a paper by Professor Bakhuyzen in *Konink. Akad. van Weten. te Amsterdam*, 1907 January 26; see *Nature*, 1907 April 4, p. 538; in which the discordance between observation and the assumption usually made is demonstrated by a concrete example.

the atmosphere; ABO the direction of the star's light, refracted once only at the boundary B. The star is thus seen in the direction OBD, instead of in the direction BA.

If $\angle ABE = \phi$, $\angle OBC = \psi$, $\angle BOL = \zeta$,

then by the law of refraction

$$\sin \phi = \mu \sin \psi \quad . \quad . \quad . \quad . \quad (1)$$

and from the triangle OBC

$$\sin \psi = \frac{OC}{BC} \sin \zeta = \frac{1}{1+z} \sin \zeta \quad . \quad . \quad . \quad (2)$$

if z be the ratio of the height of the atmosphere to the Earth's radius.

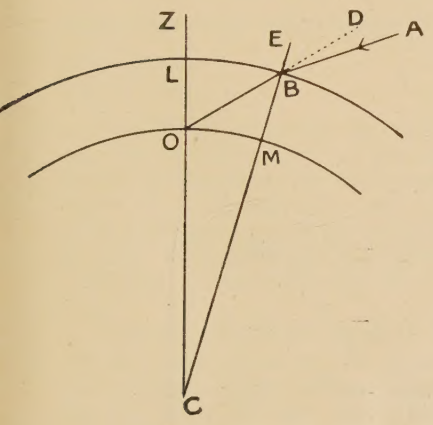


FIG. 1.

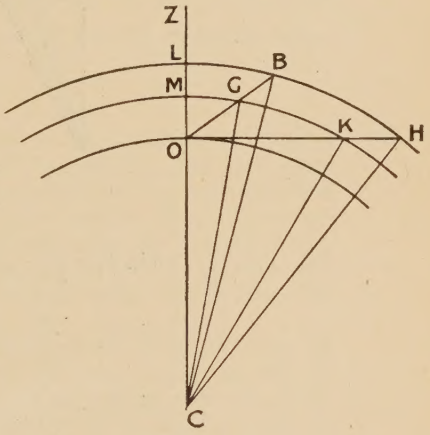


FIG. 2.

Hence $$\sin \phi = \frac{\mu}{1+z} \sin \zeta \quad . \quad . \quad . \quad . \quad (3)$$

4. The similarity of equations (2) and (3) suggests a geometrical construction for the refraction which is obvious enough, but which I do not remember to have seen in print.

With radius $\frac{1+z}{\mu}$ describe the circle MGK round C as centre (fig. 2).

Then $$\frac{\sin OGC}{\sin GOL} = \frac{1}{CG} = \frac{\mu}{1+z}.$$

Hence $OGC = \phi$, just as $OBC = \psi$, and the refraction is $\phi - \psi = GCB$.

5. If now we take the case of two homogeneous shells of atmosphere, we see that the refraction will be represented by the sum

of two angles G_1CB_1, G_2CB_2, with a gap between; the radii CG_1, CB_1, etc. being of constant length, and the angle ZOG being ζ, the apparent zenith distance, and generally, if there are n layers, expressing all the angles of incidence and refraction in terms of ζ, we have—

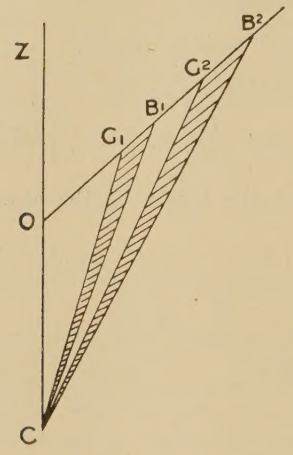


FIG. 3.

$$\left.\begin{aligned}
\sin\psi_1 &= \frac{1}{1+z_1}\ \sin\ \zeta = \frac{\mu_1}{\mu_1(1+z_1)}\sin\zeta \\
\sin\phi_1 &= \frac{\mu_1}{\mu_2}\ \sin\ \psi_1 = \frac{\mu_1}{\mu_2(1+z_1)}\sin\zeta \\
\sin\psi_2 &= \frac{1+z_1}{1+z_2}\sin\phi_1 = \frac{\mu_1}{\mu_2(1+z_2)}\sin\zeta \\
\sin\phi_2 &= \frac{\mu_2}{\mu_3}\ \sin\psi_2 = \frac{\mu_1}{\mu_3(1+z_2)}\sin\zeta \\
&\ \ \vdots \\
\sin\psi_n &= \frac{\mu_1}{\mu_n(1+z_n)}\sin\zeta \\
\sin\phi_n &= \frac{\mu_1}{\mu_{n+1}(1+z_n)}\sin\zeta
\end{aligned}\right\} \quad . \quad (4)$$

The notation is tolerably obvious: the radii of the successive surfaces of separation are $1+z_1$, $1+z_2$, $1+z_3 \cdots 1+z_n$; and the refractive indices of the successive homogeneous shells are μ_1, $\mu_2 \cdots \mu_n$. If the n^{th} be the outermost, $\mu_{n+1}=1$.

6. It will be seen that although B_1 (fig. 3) lies on the boundary of the first shell, B_2 does not lie on the boundary of the second. The radius CB_2 is not $1+z_2$ but $\frac{\mu_2}{\mu_1}(1+z_2)$; and if the refractive index decreases with the height, this is less than $1+z_2$.

Recent meteorological experiments, however, suggest that there may be anomalies in the upper strata. If the refractive index for the r^{th} layer should increase with height instead of decreasing, then CG_r would be greater than CB_r, and the angle G_rCB_r would be subtractive.

7. It is of interest to consider whether there are always gaps between the elements. If the angle B_1CG_2 vanishes, then $\phi_2=\psi_1$, or

$$\mu_1(1+z_1)=\mu_3(1+z_2) \quad . \quad . \quad . \quad . \quad (5)$$

In this case the refractions of the two assumed layers are exactly equivalent, at all zenith distances, to that of a single layer with outer radius $CB_2=\frac{\mu_2}{\mu_1}(1+z_2)$, and of refractive index $\frac{CB_2}{CG_1}=\frac{1+z_2}{1+z_1}$.

8. Condition (5) may be regarded as an equation determining μ_3, the refractive index of the shell surrounding the two inner ones; and hence we see that, *given any two homogeneous spherical shells of atmosphere with any refractive indices, they may be replaced for purposes of astronomical refraction by a single homogeneous shell, provided the refractive index of the medium immediately surrounding them has a certain value, and in that case only.*

This case must be carefully distinguished from the one in which a really homogeneous shell is arbitrarily treated as two by an imaginary surface of separation, at which there is no refraction. In this latter case one of the angles G_1CB_1 or G_2CB_2 would be zero; not necessarily the gap B_1CG_2.

9. Attention is drawn to the possibility above italicised, because it seemed at first that it might afford the explanation of the curious fact that the rough hypothesis of two or three homogeneous shells of air can give so good an approximation to our observed refractions as below. If it is possible to replace two elements by a single one, and to continue the process, there is nothing unreasonable in the exact representation of a large number of elements by a simple equivalent. But the numerical values obtained do not support this view, the gaps being wide compared with the thickness of the shells.

10. We now turn to numerical evaluation of the constants. The law of refraction is simple enough at moderate zenith distances, and we shall be chiefly concerned with its form near the horizon. In that case ϕ, ψ, and ζ are all nearly 90°, and it is more convenient to deal with their complements, which are small angles. Let us therefore write

$$\zeta=90^\circ-A, \quad \phi=90^\circ-\nu, \quad \psi=90^\circ-\lambda.$$

Let α and β be the values of λ and ν for $\zeta = 90^\circ$ or $A = 0^\circ$. Then from equations (2) and (3),

$$\cos\alpha = \frac{1}{1+z}, \quad \cos\beta = \frac{\mu}{1+z} \quad . \quad . \quad . \quad (6)$$

and equations (2) and (3) may now be written,

$$\cos\lambda = \cos\alpha \cos A, \quad \cos\nu = \cos\beta \cos A \quad . \quad . \quad (7)$$

In fig. 2, $\angle OCH = \alpha$ and $\angle OCK = \beta$: $\angle KCH = \alpha - \beta$. The refraction is $r = \phi - \psi = \lambda - \nu$.

11. In terms of the method usually adopted, we should find our constants from what we could learn of μ and z. From the point of view of this note, we are to determine α and β empirically from observed refractions at two different zenith distances; and the most convenient Z.D.'s to select are 45° and 90°. We shall confine our attention to mean refractions; and it is convenient to have the horizontal refraction exactly 33′, for a reason which will become clear when we deal with three separate layers. The mean Pulkowa refractions, as shown in the second column of Table I., have therefore been multiplied by a constant. Hence one equation between α and β is

$$\alpha - \beta = 33' = \cdot 00960 \quad . \quad . \quad . \quad (8)$$

12. At altitude 45° the equations may be combined to obtain the usual approximate law of tan Z.D. Subtracting one of equations (8) from the other, we have

$$(\cos\nu - \cos\lambda) = (\cos\beta - \cos\alpha)\cos A,$$

or

$$2 \sin\frac{\lambda-\nu}{2} = (\cos\beta - \cos\alpha)\cos A \operatorname{cosec}\frac{\lambda+\nu}{2}.$$

As a first approximation, we can neglect the differences between λ, ν, and A on the right, and put $\frac{\lambda+\nu}{2} = A = 90^\circ - \zeta$. If the refraction be denoted by r, we have for moderate zenith distances

$$r = 2 \sin\frac{\lambda-\nu}{2}.$$

$$\therefore \quad r = (\cos\beta - \cos\alpha)\tan\zeta.$$

So that $(\cos\beta - \cos\alpha)$ is the well-known "coefficient of refraction" at moderate zenith distances: and when $\zeta = 45^\circ$, $\tan\zeta = 1$, and

$$\cos\beta \mp \cos\alpha = 55''\cdot 70 = \cdot 0002700 \quad . \quad . \quad (9)$$

Hence

$$2 \sin\frac{\alpha-\beta}{2} \sin\frac{\alpha+\beta}{2} = \cdot 0002700.$$

13. Substituting the value of $\alpha - \beta$ from equation (9), we find

$$\tfrac{1}{2}(\alpha+\beta) = 1^\circ\ 36'\ 43'',$$

and since

$$\tfrac{1}{2}(\alpha-\beta) = \quad 16\ 30$$

$$\alpha = 1^\circ\ 53'\ 13'' \qquad \beta = 1^\circ\ 20'\ 13'' \quad . \quad . \quad (10)$$

14. We can now estimate the error of the approximation above, where A is substituted for $\frac{\lambda+\nu}{2}$. At $A = 45^\circ$

$$\log\cos\nu = \log\cos 45^\circ - \cdot 000118$$
$$\log\cos\lambda = \log\cos 45^\circ - \cdot 000236$$
$$\therefore\quad \nu - 45^\circ = 56'',\quad \lambda - 45^\circ = 1'\ 52''.$$

These differences are found at once from a table of log cosines. Hence

$$\frac{\lambda+\nu}{2} = 45^\circ\ 1'\ 24'',$$

and the accurate equation to replace equation (9) is

$$\cos\beta - \cos\alpha = \cdot 0002700 \times 1\cdot 00041$$
$$\tfrac{1}{2}(\alpha+\beta) = 1^\circ\ 36'\ 43'' + 1''\cdot 9$$
$$= 1^\circ\ 36'\ 45'' \text{ say},$$
$$\therefore\quad \alpha = 1^\circ\ 53'\ 15'' \quad \beta = 1^\circ\ 20'\ 15'' \quad . \quad . \quad (11)$$

15. If a table of logarithms of cosines to seconds of arc is not available, we can throw the equation

$$\cos\lambda = \cos\alpha\ \cos A$$

into the form

$$\frac{\cos A - \cos\lambda}{\cos A + \cos\lambda} = \frac{1-\cos\alpha}{1+\cos\alpha},$$

or

$$\tan\frac{\lambda - A}{2}\cdot\tan\frac{\lambda + A}{2} = \tan^2\frac{\alpha}{2}.$$

Let

$$\tan\frac{\lambda - A}{2} = \frac{\rho_1}{2}:\ \text{then since}\ \frac{\lambda + A}{2} = A + \frac{\lambda - A}{2}$$

$$\rho_1(\rho_1 + 2\tan A) = 2\tan^2\frac{\alpha}{2}(2 - \rho_1\tan A).$$

$$\therefore\quad \rho_1 = -a \pm \sqrt{a^2+4b}$$

$$= \frac{4b}{a + \sqrt{a^2+4b}},\ \text{the upper sign being the}$$

only one to take; where $a = \tan A\sec^2\frac{\alpha}{2}$, $b = \tan^2\frac{\alpha}{2}$

$$\therefore\quad \rho_1 = \frac{2(1-\cos\alpha)}{\tan A + \sqrt{\tan^2 A + \sin^2\alpha}} \quad . \quad . \quad (12)$$

When we neglect sin α in comparison with tan A, the denominator becomes 2 tan A, and

$$\rho_1 = (1 - \cos\alpha)\cot A.$$

Similarly,

$$\rho_2 = (1 - \cos\beta)\cot A,$$

and $$r = \rho_1 - \rho_2 = (\cos\beta - \cos\alpha)\cot A \text{ as before.}$$

When sin α is small compared with tan A we have

$$\rho_1 = (1 - \cos\alpha)\cot A\left[1 - \tfrac{1}{4}\frac{\sin^2\alpha}{\tan^2 A}\right] \quad . \quad . \quad (13)$$

and when $A = 45^\circ$

$$\rho_1 = (1 - \cos\alpha)(1 - \tfrac{1}{4}\sin^2\alpha)$$

$$= \frac{\alpha^2}{2} - \frac{\alpha^4}{6} \text{ to the second order in } \rho_1.$$

Similarly, $\rho_2 = \frac{\beta^2}{2} - \frac{\beta^4}{6}$; and to this order of approximation ρ_1 and ρ_2 are equal to their tangents. Hence the refraction is

$$\rho_1 - \rho_2 = (\alpha - \beta)\cdot\frac{\alpha+\beta}{2}\left[1 - \tfrac{1}{3}(\alpha^2 + \beta^2)\right] \quad . \quad . \quad (14)$$

Since $\alpha - \beta$ is the horizontal refraction we have

$$k = \frac{\text{Refraction at } 45^\circ}{\text{Refraction at Horizon}} = \frac{\alpha+\beta}{2}\left[1 - \tfrac{1}{3}(\alpha^2 + \beta^2)\right] \quad . \quad (15)$$

16. Let us now consider what happens if we have n concentric shells characterised by

$$(\alpha_1\ \beta_1),\ (\alpha_2\beta_2) \ldots (\alpha_n,\ \beta_n).$$

In each case the horizontal refraction is $(\alpha_r - \beta_r)$; so that the total horizontal refraction is

$$(\alpha_1 - \beta_1) + (\alpha_2 - \beta_2) + (\alpha_3 - \beta_3) + \ \ldots\ + (\alpha_n - \beta_n) = 33'.$$

Now the division of our shells is somewhat arbitrary. We can make them all of equal thickness, for instance ; or we can choose their relative thicknesses so that they have equal refractions at some specified zenith distance. There is a convenience in making their *horizontal refractions all equal*, so that

$$\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = \alpha_3 - \beta_3 = \ \ldots\ = \alpha_n - \beta_n = \frac{1}{n}(33') \quad . \quad (16)$$

Proceeding to the refraction at 45°, it is

$$\sum(\alpha_r - \beta_r)\left(\frac{\alpha_r + \beta_r}{2}\right)\left[1 - \tfrac{1}{3}(\alpha_r^2 + \beta_r^2)\right]$$

and hence substituting for $\alpha_r - \beta_r$, we have

$$nk = \sum\left(\frac{\alpha_r + \beta_r}{2}\right)\left[1 - \tfrac{1}{3}(\alpha_r^2 + \beta_r^2)\right] \quad . \quad . \quad (17)$$

where k denotes, as in equation (15), the ratio of refraction at 45° to that at the horizon.

One Shell.

17. The numerical results for one shell (with $\alpha = 1^\circ\ 53'\ 15''$, $\beta = 1^\circ\ 20'\ 15''$, as in equation (11)) are shown in the third column of Table I., and the discordances from the Pulkowa refractions in the fourth column. It is clear that, though the accordance is close as far as Z.D. 60°, the hypothesis does not fit the facts at all. The discordance at Z.D. 89° is nearly 5′, or 20 per cent.

TABLE I.

Z.D.	Pulkowa Refraction.	Calculated one Shell.	O—C.	Calculated Two Shells. Inner.	Outer.	Sum.	O—C.
°	″	″	″	″	″	″	″
45	55·7	55·7	0·0	9·5	46·2	55·7	0·0
60	96·2	96·2	0·0	16·3	79·9	96·2	0·0
70	151·9	152·4	− 0·5	26·0	126·0	152·0	−0·1
80	305·5	311·8	− 6·3	53·5	253·5	307·0	−1·5
81	337·5	346·0	− 8·5	59·7	280	339·7	−2·2
82	376·5	388	− 11·5	66	312	378	−2
83	424·8	442	− 17	77	352	429	−4
84	486·1	512	− 24	89	402	491	−5
85	566·2	606	− 40	107	466	573	−7
86	674·2	738	− 64	133	549	682	−8
87	825·7	935	−109	177	654	831	−5
88	1049·0	1239	−190	260	792	1052	−3
89	1397·0	1678	−281	470	925	1395	+2
90	1980·0	1980	0	990	990	1980	0

In column for one shell,

$\alpha = 1^\circ\ 53'\ 15''$ $\quad \log\cos\alpha = 9{\cdot}9997643$

$\beta = 1\ \ 20\ \ 15$ $\quad \log\cos\beta = 9{\cdot}9998817$

In columns for two shells,

$\alpha_1 = 0^\circ\ 41'\ 0''$ $\quad \log\cos\alpha_1 = 9{\cdot}9999691$

$\beta_1 = 0\ \ 24\ \ 30$ $\quad \log\cos\beta_1 = 9{\cdot}9999890$

$\alpha_2 = 2\ \ 48\ \ 50$ $\quad \log\cos\alpha_2 = 9{\cdot}9994760$

$\beta_2 = 2\ \ 32\ \ 20$ $\quad \log\cos\beta_2 = 9{\cdot}9995735$

Two Shells.

18. With two shells we have at first sight two more disposable constants; really only one. One of them is used up in the

assumption that the horizontal refractions of the two shells are equal, as in § 16. Our four constants $\alpha_1, \beta_1, \alpha_2, \beta_2$ are therefore to be determined by the equations

$$\alpha_1 - \beta_1 = \alpha_2 - \beta_2 = 16\tfrac{1}{2}' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$2k = \frac{\alpha_1 + \beta_1}{2}\left[1 - \tfrac{1}{3}(\alpha_1^2 + \beta_1^2)\right] + \frac{\alpha_2 + \beta_2}{2}\left[1 - \tfrac{1}{3}(\alpha_2^2 + \beta_2^2)\right]. \quad (19)$$

and one more condition still to be imposed. Inspection of the discordances for one shell shown in Table I. suggests that we should reduce the largest discordance to zero, *i.e.* should satisfy the observed refraction at 89°. This can only be done by "trial and error" methods. It was found useful to form tables of cos α cos A for values of A = 1°, 2°, 3° . . . 10°; and for every minute of arc of α. By their use, successive approximations gave the values

$$\alpha_1 = 0^\circ\ 41'\ 0'' \qquad \alpha_2 = 2^\circ\ 48'\ 30''$$
$$\beta_1 = 0\ \ 24\ \ 30 \qquad\quad \beta_2 = 2\ \ 32\ \ 0$$

as a close approximation; the horizontal refraction of 33′ being divided into two of $16\frac{1}{2}'$.

19. We must now examine how far these values satisfy the equation (19). They are close enough for substitution in the square terms. The following small table facilitates this substitution: the unit is ·00001.

TABLE II.

Value of $\frac{1}{3}\alpha^2$.

	10′.	20′.	30′.	40′.	50′.	60′.
0°	1	1	2	3	6	10
1°	14	18	23	28	34	41
2°	48	56	64	73	82	91
3°	101	112	124	136	149	162
4°	176	190	205	221	237	254
5°	271	289	307	326	346	366

Equation (19) runs

$$3^\circ\ 13'\ 25'' = 32'\ 45''\ (1 - \cdot00005) + 2^\circ\ 40'\ 15''\ (1 - \cdot00146)$$
$$= 3^\circ\ 13'\ 0'' - 0''\cdot1 - 14''\cdot0.$$

Hence our right-hand side is defective by 39″. Throwing this on to the second shell, which is less effective at small altitudes, we take as a trial

$$\alpha_1 = 0^\circ\ 41'\ 0'' \qquad \alpha_2 = 2^\circ\ 48'\ 50''$$
$$\beta_1 = 0\ \ 24\ \ 30 \qquad\quad \beta_2 = 2\ \ 32\ \ 20$$

With these values of the constants the refractions in columns 5 and 6 of Table I. were calculated; and it will be seen from column 8 that the error of their sum is small.

20. The question left outstanding is the choice of slightly different constants which shall perhaps satisfy the observations at (say) Z.D. 80°–87° rather better, at the expense of Z.D.'s 88°–90°, where observations are rare and possibly affected by errors of several seconds. In Table III. are given the effects of altering α_1 and β_1 by 10′, and the similar effects of altering α_2 and β_2 by 10′. It will be seen that we can, by increasing the constants for one layer, and diminishing those for the other by an exactly equal amount, leave the moderate Z.D.'s almost unaffected, and at the same time introduce sensible modifications into the large Z.D.'s.

TABLE III.

Change for 10′ Alteration of Constants.

Z.D.	Inner Shell.	Outer Shell.	Difference.	Error from Table I.
45°	(″3)	(″3)	″0·015	(″0·0)
60	(5)	(5)	0·065	(0·0)
70	(8)	(8)	0·25	(0·0)
80	(16)	(15)	2·2	−1·5
81	(17)	(16)	2·9	−2
82	20	18	2	−2
83	23	20	3	−2
84	26	22	4	−5
85	32	23	9	−7
86	40	24	16	−8
87	52	22	30	−5
88	74	18	56	−3
89	100	7	93	+2
90	0	0	0	0

21. The first five differences in the fourth column of Table III. were formed by a special method, as that used for the later ones is unsuitable. We may for moderate Z.D.'s simply differentiate the equation

$$\cos \lambda = \cos \alpha \cos A.$$

Thus

$$\frac{d\lambda}{d\alpha} = \frac{\sin \alpha \cos A}{(1 - \cos^2\alpha \cos^2 A)^{\frac{1}{2}}}$$

$$= \frac{\sin \alpha}{(\tan^2 A + \sin^2\alpha)^{\frac{1}{2}}}.$$

The refraction due to a thin shell between α and $\alpha + \delta\alpha$ at altitude A

is $\frac{d\lambda}{da}.\delta a$. The change in this for a change Δa in a is $\frac{d^2\lambda}{da^2}.\delta a.\Delta a$: where

$$\frac{d^2\lambda}{da^2} = \frac{\tan^2 A \cos a}{(\sin^2 a + \tan^2 A)^{\frac{3}{2}}};$$

and when a is small compared with A

$$\frac{d^2\lambda}{da^2} = \cot A\,[1 - \tfrac{1}{2}(1 + 3\cot^2 A)\sin^2 a].$$

If we have two shells with characteristics a_1 and a_2, the difference in effect will be proportional to

$$\tfrac{1}{2}\cot A\,(1 + 3\cot^2 A)\,(\sin^2 a_2 - \sin^2 a_1).$$

The value of the coefficient $\frac{1}{2}\cot A + \frac{3}{2}\cot^3 A$ is given in the following small table.

TABLE IV.

Value of $C = (\frac{1}{2}\cot A + \frac{3}{2}\cot^3 A)$ and $D = 0''{\cdot}00756\,C$.

Z.D.	Alt.	C.	D.
°	°		″
45	45	2·00	0·015
60	30	8·6	0·065
70	20	32·6	0·25
80	10	286	2·2
81	9	380	2·9
82	8	545	4·1
83	7	813	6·1
84	6	1295	9·8
85	5	2240	16·9

The factor for the coefficient C in our special case is

$$10' \times \text{arc of } 16\tfrac{1}{2}' \times (\sin^2 2^\circ 40' - \sin^2 0^\circ 33') = 0''{\cdot}00756.$$

The values of D for Z.D. 82°–85° are larger than those found directly in Table III., probably because the approximation used in Table IV. is ceasing to be accurate; but the quantities are of the same order of magnitude.

22. Table III. makes it clear that we cannot sensibly improve the accordance at 85° without introducing a much larger error at 89°. Every 1″ of alteration at 85° gives 10″ at 89°. Hence we may accept the constants used in Table I. as giving as good a representation of the Pulkowa refractions as we can fairly get with two shells of equal horizontal refraction.

Three Shells.

23. We divide the horizontal refraction equally into three of 11′. It is an obvious convenience to have whole minutes of arc, and for this reason the horizontal refraction was taken as exactly 33′ in the first instance. We might have taken 36′, which divides by 2, 3, and 4; but some work had already been done with 33′ before it was found that two shells would give such a good approximation.

After some few trials, the following values were found to be near the truth :—

$\alpha_1 = 0^\circ\ 29'\ 0''$	$\alpha_2 = 1^\circ\ 24'\ 0''$	$\alpha_3 = 3^\circ\ 14'\ 0''$
$\beta_1 = 0\ 18\ 0$	$\beta_2 = 1\ 13\ 0$	$\beta_3 = 3\ 3\ 0$
$\frac{1}{2}(\alpha_1+\beta_1) = 0\ 23\ 30$	$\frac{1}{2}(\alpha_2+\beta_2) = 1\ 18\ 30$	$\frac{1}{2}(\alpha_3+\beta_3) = 3\ 8\ 30$

Hence equation (17) becomes in this case

$$4^\circ\ 50'\ 8'' = 23'\ 30''\ (1 - \cdot00003) + 1^\circ\ 18'\ 30''\ (1 - \cdot00034) + 3^\circ\ 8'\ 30''\ (1 - \cdot00191)$$

$$= 4^\circ\ 50'\ 30'' - 0''\cdot04 - 1''\cdot5 - 21''\cdot6$$

showing that the adjustment has been properly made.

TABLE V.

Three Homogeneous Shells.

Z.D.	Inmost Shell.	Middle Shell.	Outmost Shell.	Sum = C.	O—C.	Difference for 10′. Inmost.	Middle.	Outmost.
45°	4″·47	15″·09	36″·14	55″·70	0″·00	1″·95	1″·85	1″·85
60	7·67	26·16	62·42	96·25	0·00	3·56	3·18	3·18
70	12·21	41·34	98·27	151·82	+0·1	5·30	5·05	5·05
80	25·3	85·0	196·0	306·3	−0·8	11·3	10·2	9·1
81	28·3	94·4	215·8	338·5	−1·0	12·2	11·1	10·2
82	31·7	106·0	239·7	377·4	−0·9	13·9	12·5	10·9
83	36·2	120·7	269·0	425·9	−1·1	16·2	14·4	11·6
84	42·4	140·5	305·5	488·4	−2·3	18·5	16·1	11·8
85	51·0	167·0	350·5	568·5	−2·3	22·3	18·6	12·5
86	63·5	205·5	407·0	676·0	−1·8	27·7	21·8	13·2
87	85·0	264·0	477·5	826·5	−0·8	39·0	28·5	11·5
88	125·5	362·0	556·5	1044·0	+5·0	52·3	28·6	8·2
89	236·5	525·0	629·0	1390·5	+6·5	83·2	19·5	2·3
90	660·0	660·0	660·0	1980·0	0·0	0·0	0·0	0·0

24. The results of this hypothesis are shown in Table V., and from the seventh column, O—C, it will be seen that there is fair accordance with the Pulkowa refractions. This accordance might be improved; by use of the differences given in the last

three columns we could form linear equations to find small corrections to the mean radii of the three shells. But the improvement would be of the nature of a compromise, and would involve—

(*a*) A study of the accuracy of our knowledge of refractions at small altitudes. We should have to settle, for instance, what error at 89° Z.D. can be made to obtain an improvement of 1″ at Z.D. 80°.

(*b*) A more accurate computation of the differences for 10′. Those given above were found in the way made clear by the following example:—

	1st diff.	2nd diff.
log cos 18′ log cos 10° = 9·9933455 = log cos (10° + 16″·0)		
	25″·3	
log cos 29′ log cos 10° = 9·9933361 = log cos (10° + 41″·3)		12″·4
	37″·7	
log cos 40′ log cos 10° = 9·9933221 = log cos (10° + 79″·0)		

The first two lines are formed to find the refraction due to the inmost layer (with constants 18′ and 29′) for altitude 10°, the result being 25″·3 as given in the 2nd column. By adding the 3rd line we find that the refraction of a shell with constants 29′ and 40′ (each 11′ greater than the former) is 37″·7; so that the difference for 11′ is 12″·4; and for 10′ would be 11″·3, which is given in the 7th column. But in forming differences of differences slight inaccuracies are multiplied, and inspection of the figures near the top of the last three columns shows that (since the numbers are nearly equal for all three shells) we are really concerned with differences of these again.

For the present, no attempt will be made to improve the approximation, and we turn to some other points.

25. It is of some interest to deduce the heights and refractive indices of the shells above found empirically.

One Shell.

$$\log(1+z) = \log \sec \alpha = 0{\cdot}0002357 = \log 1{\cdot}000544$$

$$\log \mu = \log \cos \beta - \log \cos \alpha = 0{\cdot}0001174 = \log 1{\cdot}000270.$$

The "height of the homogeneous atmosphere" is thus indicated as $4000 \times {\cdot}000544$ miles, or 2·18 miles, which is a good deal smaller than that assigned by total pressure.

This is not surprising, for in replacing a series of successive bounding surfaces by a single surface, we should expect the equivalent single surface to lie in the midst of the constituents. We can imagine, for instance, a uniform graduation which would bring the equivalent surface to the mean height, *i.e.* would give us a "height of the homogeneous atmosphere" just half that assigned by calculating the total pressure; and it would not be surprising to find that the equivalent surface should be lower than this.

Two Shells.

Equations (4) of § 5 may be put in the form

	Log.	Number.
$\log(1+z_1) = \log\ \mu_1/\mu_1 + \log\sec\alpha_1$	0·0000309	1·000071
$\log\mu_1/\mu_2 = \log(1+z_1) + \log\cos\beta_1$	0·0000199	1·000046
$\log(1+z_2) = \log\ \mu_1/\mu_2 + \log\sec\alpha_2$	0·0005439	1·001253
$\log\mu_1/\mu_3 = \log(1+z_2) + \log\cos\beta_2$	0·0001174	1·000270

If we put $\mu_3 = 1$, then $\mu_1 = 1{\cdot}000270$, $\mu_2 = 1{\cdot}000224$. The heights of the bounding surfaces in miles are $z_1 = 0{\cdot}28$, $z_2 = 5{\cdot}00$.

Three Shells.

The method of formation of the quantities has been made clear in the last case.

	Log.	Number.
$\log(1+z_1) = 0 + \log\sec\alpha_1$	0·0000154	1·000035
For $\log\mu_1/\mu_2$, add $\log\cos\beta_1$,	0·0000094	1·000022
„ $\log(1+z_2)$ „ $\log\sec\alpha_2$,	0·0001391	1·000321
„ $\log\mu_1/\mu_3$ „ $\log\cos\beta_2$,	0·0000412	1·000097
„ $\log(1+z_3)$ „ $\log\sec\alpha_3$,	0·0007331	1·001689
„ $\log\mu_1/\mu_4$ „ $\log\cos\beta_3$,	0·0001175	1·000270

If we put $\mu_4 = 1$, we have $\mu_1 = 1{\cdot}000270$, as in equation (9). This gives a check on the work.

Further, $\mu_2 = 1{\cdot}000248$, $\mu_3 = 1{\cdot}000173$. The heights of the bounding surfaces are, in miles,

$$z_1 = 0{\cdot}14, \quad z_2 = 1{\cdot}28, \quad z_3 = 6{\cdot}76.$$

It does not seem likely that these figures can have any physical interpretation. They merely emphasise the fact that a serious part of the refraction takes place near the Earth, and not high in the atmosphere. By comparing the columns for the separate shells shown in Tables I. and V. it will be seen how rapid is the increase in importance of the inner shells as we approach the horizon. In Table V., for instance, the two inner shells together only give half the effect of the outmost at moderate Z.D.'s; and they do not produce so great an effect as the outmost until we reach Z.D. 88°, when they surpass and ultimately double it in the remaining 2° or 3°. This suggests that any influence of meteorological phenomena on astronomical refractions must be sought at large Z.D.'s, which is in accordance with experience. But discussion of such points in detail cannot be undertaken at present.

Summary.

§§ 1–2. Introductory.

§§ 3–6. A graphical representation of the refraction of a homogeneous spherical shell of atmosphere and of several such shells.

§§ 7–8. Condition that two or more such shells should be replaceable by an equivalent single shell.

§§ 10–15. Empirical determination of constants for a single shell from observed refractions at Z.D. 45° and at the horizon.

§ 16. Extension of formulæ to two or more shells.

§ 17. Numerical results for one shell. The errors at 80°, 85°, and 89° are respectively $-6''$, $-40''$, $-281''$. The hypothesis cannot be said to fit the facts at all beyond Z.D. 60° or 70° at most. See Table I.

§§ 18–22. Numerical results for two shells. See Table I. The errors at 80°, 85°, and 89° are reduced to $-1''{\cdot}5$, $-7''$, and $+2''$—a very fair accordance for so obviously rough an assumption.

§§ 23–24. Numerical results for three shells: see Table V. The improvement is not great, much less marked than before; and it seems clear that the principal step was taken in passing from one shell to two.

§ 25. Some numerical values for heights, etc.

The Perturbations of Halley's Comet in the Past. Fourth Paper. The period 760 to 1066. By P. H. Cowell, M.A., F.R.S., and A. C. D. Crommelin, B.A.

We have again to acknowledge the kind assistance of Dr. Smart and Messrs. F. R. Cripps and Thos. Wright in these calculations.

In the last paper, *M.N.*, lxviii. 5, p. 378, we found March 27 for the approximate date of perihelion passage in 1066, and $44''{\cdot}686$ for the value of n at that date. For the preceding passage we used the date given by Hind (989 September 12) for the purpose of computing the perturbations, and the following results indicate that this date is correct within a few days:—

Revolution 989–1066.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°– 30°	+ ″·019	... ″	+ 532 ″
,,	330–360	− ·0035	...	...
Earth	0– 30	+ ·007	...	+ 196
,,	330–360	− ·013	...	...
Jupiter	0– 90	+ ·4535	− 531	+ 12807
,,	90–270	− ·2780	− 203	− 2905
,,	270–360	− 1·2940	+ 708	+ 22
Saturn	0– 90	− ·0560	− 23	− 1578
,,	90–270	+ ·0754	− 6	+ 1979
,,	270–360	− ·2334	+ 7	+ 47
Uranus	0–360	+ ·0030	...	+ 55
	Sums	− 1·3200	− 48	+ 11155

For this and earlier revolutions we neglect the Neptune perturbations as trifling, and take those for Venus, Earth, and Uranus from curves constructed from the results already calculated; in the case of Venus and the Earth these curves are for the combined results of the mechanical quadratures and the definite integral. Needless to say, we do not claim that the results from these curves are absolutely accurate, but such accuracy is uncalled for, since our calculated Jupiter and Saturn results are liable to sensible errors through the uncertainty attaching to our assumed position of the comet's orbit plane, etc. The resulting value of n at 989 is $44''{\cdot}686 + 1''{\cdot}320 = 46''{\cdot}006$. And calculated period in days $= \frac{1296000 - 11155}{46{\cdot}006} = 27928$. Now 1066 March 27 = J.D. 2110500; hence the calculated J.D. of previous passage is 2082572 = 989 October 9. This is 27 days later than Hind's date, a quantity sufficiently small to confirm his identification. We can reduce the discordance slightly by altering our assumed dates of perihelion passage, since the observations are not precise enough to fix the exact day. If we take them as 989 September 15 and 1066 March 25, we obtain from the revolution 989–1066 the values of n, 989 $45''{\cdot}969$, 1066 $44''{\cdot}649$; from the following revolution we obtain 1066 $44''{\cdot}688$, 1145 $44''{\cdot}920$.

The mean value for 1066 is $44''{\cdot}668$, to which corresponds $45''{\cdot}988$ in 989.

Proceeding to the revolution 912–989 we first took Hind's date, 912 April 1, and we give the perturbations deduced from this assumption:—

Revolution 912–989.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0– 30°	+″·021	...	+ 588″
,,	330–360	− ·016	...	...
Earth	0– 30	− ·0075	...	− 210
,,	330–360	+ ·0005	...	...
Jupiter	0– 90	− ·1905	+138	−5503
,,	90–270	+ ·2064	− 73	+7936
,,	270–360	+ ·0966	−598	+ 11
Saturn	0– 90	+ ·2251	− 19	+6402
,,	90–270	− ·0625	+ 15	+ 139
,,	270–360	+ ·0529	− 30	− 13
Uranus	0–360	+ ·0102	...	+ 148
	Sums	+ ·3362	−567	+9498

Taking $45''{\cdot}988$ as the value of n in 989, that in 912 is $45''{\cdot}652$, and calculated period in days $= \frac{1296000 - 9498}{45{\cdot}652} = 28181$. This gives J.D. 2054367 for the passage in 912 = 912 July 20.

This is nearly 4 months later than Hind's date, a larger quantity than is likely to arise from error in our calculation, and it is to be noticed that Hind's identification is antecedently very doubtful, being based only on observations on May 13, 15; we conclude that it is erroneous. There are vague references to other comets in 912, but nothing sufficiently precise to serve for identification. This is the first passage, reckoning back from the present time, that cannot be certainly identified with an observed comet.

Taking the calculated date 912 July 20, we obtain the following modified perturbations, 912 to 989. The value of $\int d\varpi$ may be taken the same as before.

Revolution 912–989 *revised.*

Planet.	Limits of u.	$\int dn$.	$\int d\zeta$.
Venus	0°– 30°	+″·0086	+ 24″
,,	330–360	− ·016	...
Earth	0– 30	+ ·003	+ 84
,,	330–360	+ ·0005	...
Jupiter	0– 90	− ·2445	−7026
,,	90–270	+ ·2284	+8576
,,	270–360	+ ·0966	+ 11
Saturn	0– 90	+ ·2321	+6602
,,	90–270	− ·0655	+ 69
,,	270–360	+ ·0529	− 13
Uranus	0–360	+ ·0102	+ 148
	Sums	+ ·306	+8691

This gives for the calculated period $\frac{1296000'' - 8691''}{45''\cdot682} = 28180$ days, practically the same as before.

Hence we take the date 912 July 19 as approximately correct for the perihelion passage, this result being from calculation alone, not from observation. As a check on its accuracy, we proceed to the revolution 837–912; a preliminary calculation indicates 837 February 28 for the preceding passage. It will be remembered that there is some question whether the numerous accounts of the brilliant object (or objects) that appeared in the spring of that year relate to one or to two comets. The date of perihelion passage of the earlier one, as investigated by Pingré, is indicated with tolerable precision as 837 March 1, the agreement with our calculated date being thus perfect. The other elements are also accordant except the position of the node, and it is well known that the indications of latitude in the Chinese accounts are somewhat vague, so that this is not a serious objection to the identification, especially as the comet was near the ecliptic throughout the

observations, so that a small error in the observed latitude would make a large error in the node. Hind took the second comet (if there were really two) as Halley's, finding 837 April 6 for the perihelion passage, but we think that the date March 1 is more likely to be correct.

Revolution 837–912.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$	$\int d\zeta$.
Venus	0°– 30°	+ ″·020	...″	+ 550″
,,	330–360	+ ·0015	...	...
Earth	0– 30	+ ·023	...	+ 630
,,	330–360	− ·003	...	...
Jupiter	0– 90	+ ·975	− 84	+26892
,,	90–270	− ·226	−493	− 3554
,,	270–360	−1·283	+707	+ 48
Saturn	0– 90	− ·038	+ 50	− 1040
,,	90–270	+ ·044	−132	+ 633
,,	270–360	− ·018	+ 10	+ 8
Uranus	0–360	+ ·0105	...	+ 258
	Sums	− ·494	+ 58	+24425

Taking n in 912 as 45″·682 as found above, n in 837 = 46″·176, and period in days $= \frac{1296000 - 24425}{46{\cdot}176} = 27538$, which brings us to J.D. 2026828 = 837 Feb. 25. We adopt this result as the date of perihelion passage in 837.

For the previous passage we have a fairly firm standpoint, for the comet of 760 was identified with Halley's by Laugier, from the orbit alone, before the intermediate passages had been determined, his date of perihelion passage being 760 June 11 = J.D. 1998810.

Revolution 760–837.

Planet.	Limits of u.	$\int dn$.	$\int d\varpi$.	$\int d\zeta$.
Venus	0°– 30°	+ ″·001	...″	+ 27″
,,	330–360	− ·015	...	...
Earth	0– 30	− ·006	...	− 165
,,	330–360	− ·013	...	...
Jupiter	0– 90	− ·250	−254	−7164
,,	90–270	+ ·017	+191	+7484
,,	270–360	+ ·2755	−274	− 45
Saturn	0– 90	+ ·071	− 90	+1995
,,	90–270	− ·011	+ 82	+1529
,,	270–360	− ·029	− 89	+ 1
Uranus	0–360	+ ·0145	...	+ 360
	Sums	+ ·055	−434	+4030

n 760 = 46″·176 − ″·055 = 46″·121.

Period in days $= \frac{1296000 - 4030}{46{\cdot}121} = 28013$, which brings us to J.D. $1998815 = 760$ June 15, which is 5 days later than Laugier's date.

This confirms the identity both of the comet of 760 and that of 837 with Halley's comet, and shows that we were right in taking the earlier date in 837 for the perihelion passage.

Now that the identification of the comet has been carried back to 760, it is of interest to quote the words of M. Laugier in the paper (*C.R.*, xv. p. 188) in which he identified the apparitions of 451 and 760 from the observations alone, and noticed that the average periodic time from 451 to 1378 was decidedly longer than in the subsequent seven returns:—"Le calcul des perturbations, s'il était faisable pendant cet énorme intervalle de temps, donnerait peut-être l'explication d'une variation aussi grande du temps periodique; mais, dans le cas où il ferait défaut, on pourrait se rappeler qu'une diminution analogue, quoique plus petite, a été observée dans les retours de la comète à courte periode."

He goes on to speak of the retardation of Encke's comet, and of the suggestions that were put forward to account for this, viz. resisting medium, and Bessel's suggestion of the effect of loss of matter from the comet when passing perihelion, to which we may add the more recent theory of the effect of light-pressure on finely divided matter.

The calculation of the perturbations to which M. Laugier refers has now been carried out, not with absolute rigour, but sufficiently to reconcile theory and observation within a few days in each revolution; and we have found that the large changes in the periodic time are explicable on gravitational grounds alone. Our work does not justify us in asserting that the effect of the various non-gravitational causes referred to above is absolutely insensible; but it suffices to show that their combined effect does not amount to more than a week per revolution, and it is probably still less than this.

Solar Prominences in 1907, *observed at the Kodaikánal Observatory.*
By John Evershed.

The year has been one of considerable activity as regards prominences, notwithstanding a noticeable reduction in the mean profile area, which amounts to about 10 per cent. for all classes of prominences.

At the Kodaikánal Observatory 78 prominences of 100″ or upwards have been recorded photographically and visually during 305 days of observation. The photographic records show also that large eruptive prominences have not been infrequent, seven of this class having been recorded; the greatest elevation measured was 6½ minutes of arc in a transient eruption, photographed on March 14 in solar latitude +52. A remarkable eruption was also photographed by Fox at the Yerkes Observatory on May 21 in solar latitude −68.

The general activity of the two hemispheres of the Sun compared with the previous year may be inferred from the following figures, deduced from the Kodaikánal results:—

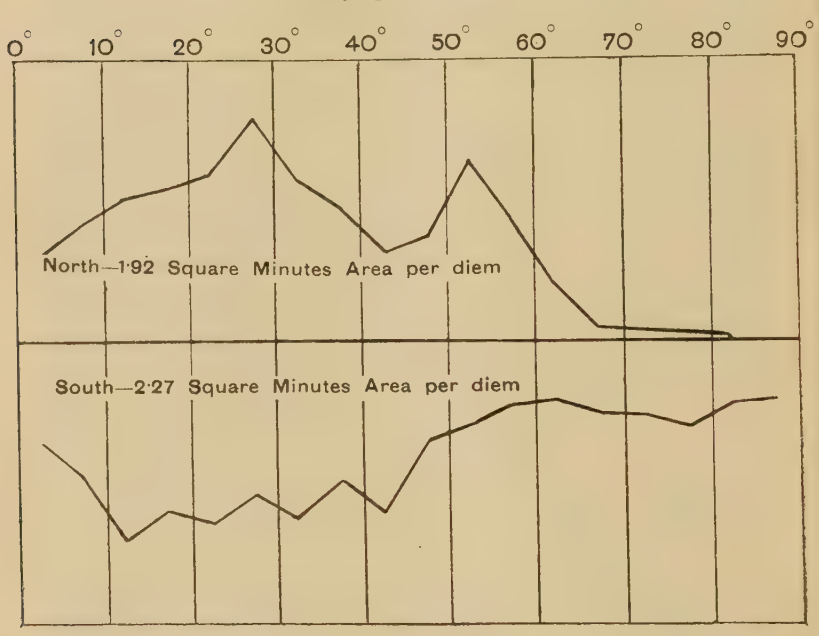
Mean Daily Profile Areas of Prominences.

	1906.		1907.	
North	2·51	square minutes.	1·92	square minutes.
South	2·17	„ „	2·27	„ „
Total	4·68	„ „	4·19	„ „

It is seen from the above that the general reduction of activity in 1907 is confined to the northern hemisphere, the southern showing a slight increase. In the latitude distribution a remarkable difference is shown between the two hemispheres, which are usually more or less symmetrical as regards the latitudes of the zones of maxima and minima. From the beginning of the year the northern polar prominences, which were strongly represented during 1906, practically ceased to exist, whilst the south polar region still continued active, the whole region between −45° and the south pole producing a very considerable number of large prominences. The region from lat. −10° to −45° has been the most prolific, however, in this hemisphere; but no clearly marked zones of maxima are shown. In the north, on the other hand, two well-defined maxima occur in the zones +25° to 30° and +50° to 55°.

Metallic prominences were of frequent occurrence, 111 having been recorded. Of these, 54 were confined to the northern spot zone, and had a mean latitude of +15·7°; 50 were confined to the southern spot zone, with a mean latitude of −15·6°; the remaining 7 were distributed in longitude in a narrow zone entirely outside the spot regions, the mean latitude being −72°. The only metallic

elements observed in these high-latitude prominences were Na, Mg, and Fe, whilst some of the prominences in spot-latitudes gave in addition the lines of Ba and Ca, together with a considerable number of unidentified lines, probably including Ni, Mn, Cr, and Ti.


Heliographic Latitude.
0° 10° 20° 30° 40° 50° 60° 70° 80° 90°
North—1·92 Square Minutes Area per diem
South—2·27 Square Minutes Area per diem

Distribution Curve of the Prominences for 1907.

The ordinates give the mean daily profile areas for each zone of 5°, obtained fro observations on 296 days.

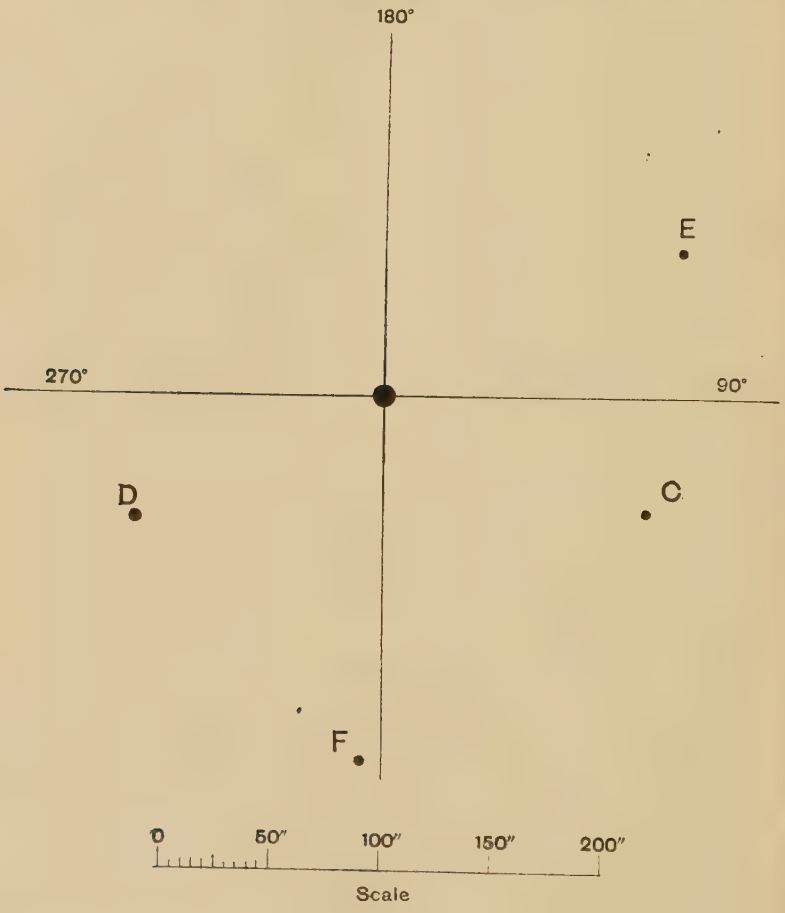
The Proper Motion of Small Stars. By S. W. Burnham.

Small stars having any sensible proper motion which can be detected by meridian or micrometric observations, so far placed on record, are much rarer than parallax stars taken from all magnitudes, bright and otherwise. Very few examples have been detected down to this time, although more or less searched for by all double-star observers and others using the micrometer for determining the relative positions of stars. The small stars referred to may be generally classed as below the limiting magnitude of the Durchmusterung, or from the tenth magnitude to the faintest stars which have been accurately measured by large instruments. Of course it is to be expected that this apparent fixity in space would be found in the great majority of instances; or that the motion would be so small that it could not possibly be separated from the unavoidable errors of observation in the most careful measures by all astronomical instruments. Therefore these small stars furnish by far the best means of determining any change in the position of brighter stars when the distance is within reach of the micrometer, and give the proper motion of the large stars more accurately than any number of positions with the meridian circle from the beginning of such observations, provided the interval of time covered by the micrometer measures is sufficient to practically eliminate the smaller errors of that instrument. So, when the prominent stars have been connected by measures with faint stars in the field by the Struves and other old observers, we have a value of the proper motions of the large stars which is a safe and certain correction to that given by meridian positions.

It is hardly necessary to say in this connection that the faint stars referred to have nothing to do with a large and distinct class of these objects which are moving in space with much brighter stars, and at exactly or nearly the same rate, and in substantially parallel directions. We have a large number of these attending stars, some of them of the smallest magnitudes visible in the largest refractors, and in brightness anywhere between that and the naked-eye star. In many instances the moving stars are separated by several minutes of arc. Whether these constitute binary systems in the ordinary sense of the term, or what common proper motion of stars so remote from each other implies, is, at this time, a matter of speculation only, and must necessarily remain so until careful observations have been made, extending over possibly several hundreds of years. That these stars have something in common seems to be at least highly probable. A large number of systems of this class will be found in my *General Catalogue of Double Stars*, indexed in Part I. under "Common Proper Motion," and "Stars of the 61 Cygni type."

One of the very few examples I have been able to find, and

perhaps the only one worth mentioning, of an isolated small star having a decided proper motion, is a faint star of about the twelfth magnitude in a low-power field with 17 *Lyræ* (Σ 2461). The observations of this star are sufficient in point of time to show that it has a remarkable movement, and one without any parallel in



17 *Lyræ.*

stars of the lower magnitudes, so far as appears from reliable observations of any kind.

The diagram shows the positions of this and other small stars which have been measured from 17 *Lyræ.*

The magnitudes of the three smaller stars have been measured photometrically by Mr. F. C. Jordan of the Yerkes Observatory.

The magnitude of the large star D, which is DM 32°, 3325, is assumed to be 9·0.

C = 12·00 mag.
E 10·96
F 10·87

My measures of these stars with the 40-inch are :—

A and C.

1905·717	66°·4	127″·37
5·720	66·4	127·73
5·736	66·5	127·66
6·906	66·1	129·53
6·964	66·1	129·46
7·258	66·1	130·01
7·271	66·2	130·14
7·291	66·3	130·38

A and D.

1906·906	295°·5	126″·51
6·964	295·6	126·51
7·258	295·7	126·03
7·271	295·3	126·32
7·291	295·7	126·61

A and E.

1906·906	116°·5	150″·93
6·904	116·6	150·77
7·258	116·9	150·86
7·271	116·7	150·85

A and F.

1906·906	356°·7	167″·54
6·964	356·8	167·47
7·271	357·2	167·71

For the relation of A and C we have the following earlier positions by Engelhardt :—

1887·81	72°·02	102″·28	1n	Eng.
1894·85	69·23	112·31	2n	Eng.
1905·72	66·40	127·59	3n	*β*
1907·14	66·16	129·90	5n	*β*

These positions give for the most probable apparent annual motion of C :—

1″·542 in 45°·9.

The large star, 17 *Lyræ*, has a small proper motion which is given from meridian observations :—

0″·112 in 68°·5	Auwers.
0·121 in 74·8	Newcomb.

A more accurate value of this movement can be found from the measures of the other distant stars which it is certain have no sensible proper motion. These positions are :—

A and D.

1887·73	296°·40	124″·34	2n	Eng.
1894·85	296·00	125·32	2n	Eng.
1905·72	295·70	126·12	1n	β
1907·14	295·56	126·40	5n	β

A and E.

1887·81	116°·00	152″·65	1n	Eng.
1894·85	116·37	152·20	1n	Eng.
1905·72	116·50	150·75	1n	β
1907·10	116·67	150·85	5n	β

A and F.

1887·73	357°·77	168″·20	2n	Eng.
1894·85	357·50	168·11	2n	Eng.
1907·05	356·90	167·57	3n	β

These measures give for the proper motion of 17 *Lyræ* :—

	R.A.	Decl.
AD	+0″·1263	+0″·0342
AE	0·1260	0·0224
AF	0·1307	0·0402
Mean	0·1277	0·0323

Therefore the corrected proper motion is :—

0″·132 in 75°·8.

Applying this to the apparent movement of C, we have for the real proper motion of that star :—

1″·658 in 48°·2.

A photographic plate of this field was taken 1907·444 with the 40-inch. With a Wallace colour screen, and an exposure of about fifteen minutes, the resulting negative was beautifully sharp and well defined, and practically perfect for measuring the relative positions of the several stars.

As direct measurement of the position-angle cannot be made with any instrument here, the measures of this plate were confined to distances. The mean results of six sets of measures by four observers are as follows:—

AC	= 130″·58
AD	126·40
AE	150·92
AF	167·78
CE	121·72

A study of the details of these measures seems to show that they have about the same accuracy as those made under the best conditions with the micrometer, and that the uncertainty of the result is probably less, or not more, than 0″·2.

Whether a proper motion star of this magnitude is more likely to have a parallax large enough for certain measurement, which should be at least 0″·3, by either or both methods, than a twelfth magnitude star taken at random, is a matter to be determined hereafter, but it would not be unreasonable to presume that such a star might be nearer our system.

There is one other example of proper motion in a small star, but nearly three magnitudes brighter than the one already referred to, which was discovered by Dr. Max Wolf in his photographic researches (*A.N.*, 4101) by the change shown on plates made in 1892 and 1906. This is a 9·2*m* star, not given in the *D.M.*, the place (1900) of which is—

R.A. = 11h 23m 20s.1
Decl. = 8° 6′ 1″

There is a similar star, given by Wolf as 9·1*m*, closely preceding; and a 13*m* star more distant in the same quadrant. I have made two sets of measures of these stars as follows:—

A and B. (9·7—9·7.)

1906·906	252°·1	81″·63
7·006	252·0	81·68
7·102	252·1	81·42
7·120	252·2	81·44
1907·03	252·1	81·54
1908·118	253°·2	80″·92
·156	253·1	81·11
·159	253·1	80·68
·192	253·0	80·82
1908·15	253·1	80·88

B and C. (12·0.)

1906·906	275°·9	155″·17
7·102	275·5	154·48
7·120	275·7	154·17
1907·04	275·7	154·61
1908·118	275°·9	154″·90
·159	275·8	154·70
·192	275·7	154·50
1908·15	275·8	154·70

As Dr. Wolf gives no measures of the relations of these stars which can be utilised in this connection, I have measured as carefully as possible their positions on the paper print for comparison with the micrometer positions, with the following results :—

A and B.

1892·23	242°·1	91″·47		Wolf.
1907·03	252·1	81·54	4n	β
1908·15	253·1	80·88	4n	β

B and C.

1892·23	275·9	154″·94		Wolf.
1907·04	275·7	154·61	3n	β
1908·15	275·8	154·70	3n	β

It is evident that the comparison star B has no proper motion, and that the change shown by the measures of AB is due wholly to the proper motion of A. A comparison of the position in 1892 with a mean of the two later measures gives for the proper motion of A :—

1″·225 in 190°·2.

The minimum distance of A and B of 72″ will be reached in about thirty years, after which they will slowly separate.

The 7·5*m* star, DM 8°, 2512, which is 70ˢ following and 185″·7 south, has a 13·2*m* companion not heretofore observed :—

1907·12	285°·0	18″·37
·14	286·3	18·37

I do not find any proper motion assigned to this star.

Yerkes Observatory :
April 10.

New Double Stars. By the Rev. T. E. Espin, M.A.

No.	B.D.	R.A. 1900. h m	Decl.	P.	D.	Mags.		Nights.	Date. 1908.	
			° ′	°	″					
552	+56,143	0 47·4	+56 41	85·9	12·60	7·0	13·0	2	·015	
553	+45,405	1 33·5	45 32	144·1	5·15	8·8	9·2	3	·014	
554	+42,549	2 27·4	42 32	24·9	5·80	9·1	9·3	1	·099	
555	+41,501	32·0	41 46	323·9	9·62	8·5	11·5	2	·083	
556	+41,543	42·2	42 0	352·7	6·40	8·4	10·7	2	·073	
557	+47,712	44·3	47 41	315·8	5·04	9·1	12·4	4	·075	
558	+45,710	3 0·0	45 22	357·8	8·47	7·5	9·4	2	·082	
559	...	6·8	43 54	250·6	3·17	9·3	11·2	2	·116	
560	+45,784	26·3	45 55	140·8	8·60	8·4	10·1	3	·091	
561	+45,787	26·6	45 25	38·8	7·40	8·9	9·1	2	·087	
562	+44,769	34·0	44 17	358·4	2·91	9·3	11·7	5	·066	
563	+44,809	47·6	44 35	23·3	6·14	9·0	12·6	4	·082	
564	+42,876	56·7	42 30	115·3	2·29	8·9	9·3	4	·117	
565	+42,890	4 0·0	42 35	62·2	4·75	9·3	10·6	2	·090	
566	+46,881	18·5	46 19	281·3	5·10	9·4	12·0	3	·106	CD
				150·2	11·10	8·9	10·8	2	·097	AB
				148·0	27·15			2	·097	AC
567	+44,945	18·7	45 1	128·9	7·97	8·8	9·4	3	·106	BC
				351·8	36·32	A = 8·8		2	·097	AB
568	+42,969	20·7	42 58	304·3	5·07	7·7	12·0	2	·083	
569	+44,967	24·5	44 43	19·5	7·72	9·0	12·8	3	·122	
570	+41,898	26·8	41 15	178·7	3·90	9·1	13·0	1	·153	
571	+48,1146	39·9	49 3	59·9	3·35	9·1	9·2	2	·143	
572	+41,966	41·9	41 28	93·1	3·05	8·8	12·0	1	·153	
573	+42,1228	5 9·4	42 33	122·9	5·01	8·0	10·5	3	·040	
574	+47,1122	10·3	47 12	87·7	1±	9·7	10·3	3	·108	BC
				63·1	33·65	A = 9·4		3	·108	AB
575	+48,1264	16·9	48 16	353·6	14·35	8·1	10·7	3	·113	
576	+42,1274	17·2	42 31	342·8	8·45	8·0	13·7	2	·015	AB
				236·3	42·35		8·2	2	·015	AC
577	...	17·3	47 17	133·5	2·30	9·7	11·5	2	·101	
578	+49,1403	40·2	49 22	41·7	2·02	9·1	9·2	2	·131	
579	−47·1249	6 1·0	47 26	115·3	6·57	8·5	11·0	2	·119	BC
				341·3	57·00	A = 8·3		2	·119	AB
580	+44,1380	4·0	44 45	226·3	6·85	9·0	9·4	3	·078	AB
				119·2	22·30	C = 12·0		1	·082	AC
581	−49,1470	7·3	49 0	61·6	3·25	8·8	11·5	2	·121	
582	+44,1492	29·4	44 10	301·9	6·18	9·6	12·3	3	·107	
				79·0	33·92	A = 9·3		3	·107	
583	+44,1527	38·0	44 35	69·4	4·67	9·4	9·6	4	·082	

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.		Nights.	Date.	
		h m	° ′	°	″				1908.	
584	+47°1353	43·9	47 21	325·4	3·00	9·4	10·0	2	·045	
585	+45,1430	7 17·7	45 3	237·3	2·72	7·7	11·7	2	·138	
586	+41,1670	23·3	41 49	16·8	13·40	8·1	11·5	2	·189	
587	+46,1307	39·3	46 9	80·7	4·80	8·9	9·2	2	·083	
588	...	40·1	47 33	304·2	2·15	9·5	9·8	3	·078	
589	+48,1576	41·2	48 1	178·3	10·00	7·7	13·7	4	·132	
590	+43,1746	47·8	43 25	69·8	7·40	9·0	9·4	2	·095	
591	+45,1536	8 0·5	45 30	48·2	1·62	9·4	9·6	2	·083	
592	+41,1799	6·3	41 52	327·4	2·72	8·6	9·9	3	·308	
593	+41,1810	10·0	41 12	208·2	4·70	9·4	9·6	2	·235	BC
				230·2	19·82	A = 8·5		2	·235	AB
594	+43,1820	18·6	43 35	183·0	2·48	9·2	12·0	3	·112	
595	+48,1654	24·1	48 6	234·9	7·75	8·5	13·2	2	·082	
596	+46,1436	41·1	45 54	201·3	2·65	8·6	9·0	3	·095	
597	+45,1640	42·8	45 48	262·6	5·33	8·5	11·8	3	·091	
598	+47,1630	55·2	47 45	261·0	7·20	8·6	10·5	2	·119	
599	+41,1915	59·7	41 31	137·3	3·02	9·0	11·8	2	·152	
600	+50,1673	9 35·5	49 49	71·6	3·65	9·0	13·5	2	·152	
601	+46,1549	41·1	46 21	287·5	3·47	9·0	9·2	2	·119	
602	...	47·6	48 36	32·9	2·95	10·4	11·1	3	·261	
603	+48,1887	10 36·7	48 43	98·5	10·57	9·1	11·0	2	·260	
604	+45,1865	40·7	45 43	52·7	1·82	10·6	11·4	2	·249	
	+49,1900	49·2	48 40	158·4	47·00	8·0	8·7	2	·292	
605	+48,1953	11 27·0	48 11	64·5	4·17	8·9	13·7	2	·260	
	+43,2261	12 43·5	42 53	50·1	46·07	7·5	7·7	2	·323	
606	+43,2293	56·8	42 58	284·9	8·53	8·5	12·0	3	·304	
607	+43,2299	13 0·7	43 15	224·9	6·10	9·0	11·5	1	·334	
608	+48,2138	33·0	48 45	271·8	2·57	9·0	9·2	2	·289	
609	+48,2224	14 33·2	48 14	12·3	4·65	9·0	10·7	2	·289	

Notes.

559.—38^{s} f, 50″ S, 0Σ 51.

574.—Measures of the close pair are unsatisfactory, and I had some doubts about the star being really double. Professor Burnham has, however, kindly looked at it with the 40 in., and confirms its duplicity.

583.—A 14 mag. south.

602.—Found and measured in looking for *h* 2510.

604.—The star is so faint that it is surprising that it is in the B.D.

605.—Angle mean of 61°·2, 67°·8. This is a very difficult pair to measure, from the faintness of the *comes*.

608.—In field south of *h* 2667.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1907.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, focal length 28 feet. The power generally employed was 670. When bright stars were observed a blue glass shade was usually employed to diminish the light and irradiation. The initials in the last column are those of the observers, viz.—

L.	Mr. Lewis.	W.B.	Mr. Bowyer.
B.	Mr. Bryant.	H.F.	Mr. Furner.

The stars have been observed from a working catalogue containing all the G. W. Hough stars within the working zone, and a number of miscellaneous stars showing motion, or for which observations are required.

Some of these being wide pairs of no immediate interest, the present list of measures is confined to stars of which the separation is under 4″, or which show orbital motion.

Stars observed but not included in this List.

Struve Stars.

Σ 183 AC	Σ 1158 AB	Σ 1384	Σ 1541	Σ 2185 AB	Σ 2524
1000	1158 AC	1426 AC	1555 AC	2185 AC	2690 AB
1100	1158 AD	1468	1585	2220	2704 AD
	1158 AE	1481 AB	1612	2434	2709
	1196 AC	1481 AC	2140	2458	2725
	1196 BC	1507			

G. W. Hough Stars.

Ho. 494	Ho. 337 AB	Ho. 387	Ho. 423	Ho. 274	Ho. 597
495	21	547	269	115	604
8	28	406 AB	433	116	475 AC
11	29	552	434	586	476
316	523 AB	553	91	117	478
219	348	409	446	588 AB	191 AC
501	349	557	445	588 AC	302 BC
325	38	416	448	593	204
327	259	419	450	131	
508	543	561	451	145	

Royal Observatory, Greenwich:
1908 *May* 5.

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.		Epoch 1907.	Obs.
Hu. 405 ...	0 12	65 57	275·5	1·18	4	9·3	9·5	·902	W.B.
β 1093 ...	0 16	79 31	69·5	0·33	1	7·3	8·2	·827	B.
β 779 ...	0 23	66 55	257·3	1·04	1	8·5	9·0	·827	B.
Ho. 212 ...	0 31	94 5	245·0	0·31	1	6·0	6·0	·827	B.
Hu. 411 ...	0 33	67 30	102·7	0·65	1	8·5	8·5	·925	W.B.
Hu. 413 ...	0 42	67 15	241·7	0·80	1	8·0	9·2	·925	W.B.
Ho. 306 ...	0 43	64 56	163·7	1·19	1	8·5	8·8	·044	W.B.
Ho. 4 ...	0 46	56 33	200·4	1·57	3	9·0	9·0	·592	W.B.
Σ 73 ...	0 50	66 52	28·2	1·13	1	6·2	6·8	·827	B.
			29·9	0·85	2	...		·864	W.B.
Ho. 307 ...	0 52	58 17	86·5	2·76	1	9·5	9·7	·044	W.B.
Σ 113 ...	1 15	91 0	355·2	1·33	1	6·2	7·2	·827	B.
Ho. 9 ...	1 24	68 45	94·8	2·44	3	9·0	10·0	·592	W.B.
Σ 138 ...	1 31	82 50	33·1	1·62	1	7·3	7·3	·827	B.
Σ 158 ...	1 42	57 18	257·7	1·90	2	8·3	8·8	·864	W.B.
Σ 183 AB ...	1 50	61 39	1·6	0·45	1	7·5	8·2	·827	B.
Hu. 1033 ...	1 50	54 6	234·4	0·85	1	8·5	8·8	·044	W.B.
Σ 186 ...	1 51	88 37	35·1	0·73	1	7·2	7·2	·827	B.
Σ 208 ...	1 59	64 31	91·2	0·48	1	6·2	8·4	·058	W.B.
			78·6	0·58	1	...		·162	H.F.
Ait. 959 ...	2 13	59 9	360·7	3·84	3	9·0	12·0	·342	W.B.
Ait. 962 ...	2 16	60 28	76·6	0·53	1	8·9	9·2	·162	H.F.
Σ 305 ...	2 42	71 1	316·3	3·22	1	7·3	8·2	·104	H.F.
			315·4	3·19	3	...		·673	W.B.
Ho. 317 ...	2 52	73 7	304·3	3·46	2	8·1	11·0	·867	W.B.
Ho. 500 ...	3 6	54 15	222·1	0·68	1	8·5	9·0	·162	H.F.
Ho. 502 ...	3 9	54 36	19·5	0·77	1	8·5	9·0	·162	H.F.
Hu. 1058 ...	3 19	50 6	114·7	0·75	2	7·8	8·5	·192	H.F.
Ait. 979 ...	3 20	59 35	270·9	1·48	2	9·2	10·0	·482	W.B.
Ho. 322 ...	3 20	44 43	124·1	1·90	1	9·2	9·5	·222	H.F.
Ait. 983 ...	3 26	60 40	304·1	0·49	1	8·5	9·2	·162	H.F.
Ait 987 ...	3 37	60 30	7·6	1·09	3	9·6	9·7	·347	W.B.
Hu. 103 ...	3 39	40 25	203·1	1·01	1	8·1	8·4	·222	H.F.
Ho. 504 ...	3 39	54 26	188·3	0·98	1	7·8	8·0	·893	W.B.
Hu. 815 ...	3 51	68 29	204·7	2·45	3	8·0	11·3	·324	W.B.
Ho. 505 ...	3 53	57 30	194·9	1·23	1	8·0	10·0	·044	W.B.
Σ 483 ...	3 58	50 43	223·6	0·53	2	8·0	9·5	·192	H.F.
β 1277 ...	4 0	61 51	300·7	0·75	1	8·0	12·2	·227	L.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
β 1005 ...	4 1	61 18	64·2	2·40	1	8·5	11·7	·189	L.
β 1232 ...	4 3	61 3	188·2	0·43	2	8·4	9·3	·208	L.
Espin 278 ...	4 3	50 3	172·4	2·34	2	7·7	9·2	·192	H.F.
Ho. 326 ...	4 4	61 35	165·7	0·49	2	8·0	8·0	·208	L.
OΣ 77 ...	4 10	58 32	197·8	0·28	1	7·5	7·5	·227	L.
Ho. 15 ...	4 19	60 5	150·8	0·76	2	8·0	8·0	·492	W.B.
			151·9	0·70	1	...		·162	H.F.
β 1044 ...	4 35	73 39	219·8	0·66	1	9·0	11·0	·189	L.
Ho. 333 ...	4 38	69 55	164·9	2·75	1	9·2	9·3	·104	H.F.
Ho. 224 AB	4 59	61 24	281·5	1·70	1	9·0	10·7	·159	W.B.
Ait. 1024 ...	4 59	60 30	349·5	0·80	1	8·1	8·9	·227	L.
			355·9	0·56	2	...		·932	W.B.
Ait. 1026 ...	4 59	60 9	38·4	1·06	2	7·8	11·0	·492	W.B.
OΣ 95 ...	5 0	70 19	327·2	0·84	1	6·6	7·2	·189	L.
Ait. 1028 ...	5 1	60 3	238·6	0·45	1	8·5	9·1	·939	W.B.
Hu. 217 ...	5 24	54 42	259·0	0·61	2	7·0	8·5	·192	H.F.
Σ 749 ...	5 32	63 8	170·0	0·78	1	7·0	7·1	·175	B.
			172·7	0·97	1	...		·925	W.B.
Ait. 1040 ...	5 39	58 44	130·8	0·74	1	8·2	9·3	·222	H.F.
Ho. 337 BC	5 50	66 44	110·5	1·23	1	9·0	9·2	·159	W.B.
Ho. 23 ...	6 9	75 31	243·9	2·74	1	8·2	12·0	·159	W.B.
Ho. 239 ...	6 45	75 12	130·7	0·51	3	8·3	8·3	·177	W.B.
Ait. 1061 ...	6 54	64 49	321·3	0·40	2	8·2	8·5	·151	W.B.
Espin 289	6 57	50 54	101·8	2·04	1	9·4	9·7	·230	W.B.
Ho. 342 ...	6 58	76 46	83·7	1·39	1	8·0	8·8	·159	W.B.
Σ 1037 ...	7 7	62 37	299·2	0·52	1	7·1	7·1	·241	W.B.
Ho. 33 ...	7 15	67 40	326·4	2·95	1	9·0	12·0	·236	W.B.
Ho. 36 ...	7 42	64 19	120·6	0·57	1	8·5	8·5	·159	W.B.
Ho. 250 ...	7 52	68 48	153·1	0·54	3	7·0	9·0	·210	W.B.
Σ 1187 ...	8 4	57 31	41·9	2·23	2	7·1	8·0	·243	W.B.
Σ 1196 AB	8 7	72 4	340·3	1·21	2	5·0	5·7	·151	W.B.
			341·7	1·19	1	...		·222	H.F.
Hu. 1123 ...	8 9	53 14	155·7	0·36	2	8·5	8·8	·236	W.B.
Hu. 626 ...	8 14	57 23	153·6	3·39	1	8·0	12·0	·356	W.B.
Ho. 525 ...	8 18	69 42	150·7	0·55	3	8·5	8·5	·242	W.B.
Hu. 627 ...	8 25	55 10	264·3	1·01	1	9·0	10·0	·356	W.B.
Σ 1273 AC	8 42	83 12	241·8	3·07	1	3·8	7·8	·260	H.F.
Espin 294 ...	8 43	53 31	160·2	1·70	2	9·0	9·2	·276	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights	Mags.		Epoch 1907.	Obs.
	h m	° ′	°	″					
Hu. 861 ...	8 53	75 25	20·9	0·28	1	8·5	9·1	·241	W.B.
Hu. 718 ...	8 55	57 13	192·0	0·41	2	8·7	8·9	·299	W.B.
Σ 3121 ...	9 13	61 3	35·0	0·43	3	7·5	7·8	·242	W.B.
Ho. 42 ...	9 9	56 4	7·9	1·62	1	9·5	9·5	·255	W.B.
Σ 1333 ...	9 13	54 15	47·4	1·60	1	6·6	6·9	·323	L.
Σ 1334 ...	9 13	52 48	239·1	2·91	1	4·0	6·7	·323	L.
Ho. 43 ...	9 14	68 48	298·0	0·31	1	8·0	8·5	·230	W.B.
Ho. 364 ...	9 14	66 42	332·9	3·87	1	8·2	11·2	·219	W.B.
Σ 1348 ...	9 20	83 12	318·5	1·81	2	7·5	7·6	·225	W.B.
Σ 1355 ...	9 22	83 19	329·2	2·24	2	7·2	7·2	·282	H.F.
Σ 1372 ...	9 32	73 19	56·4	0·69	2	8·2	8·3	·282	H.F.
Σ 1374 ...	9 35	50 35	286·6	3·27	1	7·0	8·3	·296	W.B.
			288·3	2·95	1	...		·334	H.F.
Ho. 369 ...	9 46	53 5	275·5	0·37	1	7·7	7·8	·323	L.
Σ 1389 ...	9 47	62 32	306·4	1·99	2	8·0	9·0	·277	W.B.
			308·2	2·65	1	...		·260	H.F.
			307·7	2·19	1	...		·323	L.
Σ 1406 ...	10 0	58 26	228·3	0·98	1	8·0	8·7	·334	H.F.
Σ 1413 ...	10 7	73 9	96·0	2·18	2	8·9	8·9	·282	H.F.
			97·6	1·90	1	...		·296	W.B.
Σ 1420 ...	10 13	50 24	327·5	2·24	1	8·2	9·9	·296	W.B.
Hu. 875 ...	10 13	52 2	64·1	1·05	1	7·0	9·8	·356	W.B.
Σ 1423 ...	10 14	68 56	57·7	1·23	1	8·6	9·3	·304	H.F.
Σ 1426 AB	10 16	83 4	283·0	0·72	2	7·5	8·0	·282	H.F.
0Σ 218 ...	10 23	85 56	79·1	0·78	2	8·2	9·7	·282	H.F.
Σ 1439 ...	10 25	68 44	113·0	1·94	4	8·0	8·5	·238	W.B.
			114·0	1·83	1	...		·260	H.F.
Σ 1457 ...	10 34	83 46	314·8	1·28	2	7·6	8·5	·282	H.F.
Σ 1476 ...	10 44	93 30	181·7	2·18	2	7·2	8·0	·319	H.F.
Σ 1500 ...	10 55	92 56	308·1	1·24	2	7·6	8·2	·319	H.F.
Σ 1517 ...	11 9	69 19	87·1	0·34	2	7·3	7·3	·301	B.
			84·0	0·45	2	...		·370	W.B.
Σ 1523 ...	11 13	57 54	131·6	2·86	1	4·0	5·0	·296	W.B.
			131·6	2·68	1	...		·334	H.F.
Σ 1525 ...	11 14	41 58	176·8	1·64	1	9·0	9·0	·392	H.F.
Σ 1536 ...	11 19	78 58	49·2	2·21	2	3·9	7·1	·301	B.
Σ 1537 ...	11 20	68 52	355·3	2·80	1	7·6	8·6	·219	W.B.
Σ 1555 AB	11 31	61 40	353·1	0·39	2	6·4	6·8	·301	B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights	Mags.		Epoch 1907.	Obs.
	h m	° ′	°	″					
Hu. 888 ...	11 38	68 25	320·1	0·69	3	8·4	8·9	·301	W.B.
Σ 1581 ...	11 51	43 53	172·0	2·04	1	8·3	9·5	·392	H.F.
Σ 1589 ...	11 55	45 50	161·3	1·94	1	9·0	9·5	·392	H.F.
Σ 1606 ...	12 6	49 33	328·7	1·06	1	6·3	7·0	·392	H.F.
Σ 1639 ...	12 20	63 55	351·3	0·36	2	6·7	7·9	·301	B.
Σ 1643 ...	12 23	62 28	34·7	2·15	1	8·7	9·2	·348	W.B.
			37·6	2·20	1	...		·438	L.
Σ 1647 ...	12 26	79 47	224·4	1·46	3	7·5	7·8	·308	W.B.
Hu. 892 ...	12 38	74 59	163·8	1·01	2	9·3	9·3	·301	B.
Hu. 893 ...	12 43	77 13	33·9	1·55	2	9·1	9·1	·301	B.
Hu. 894 ...	12 49	76 20	144·4	1·02	2	9·1	9·1	·301	B.
OΣ 256 ...	12 52	90 24	76·6	0·60	2	7·2	7·6	·301	B.
OΣ 261 ...	13 8	57 26	346·9	1·66	2	6·9	7·4	·301	B.
OΣ 269 ...	13 29	54 38	215·6	0·35	2	6·5	7·0	·319	B.
OΣ 278 ...	14 9	45 24	82·5	0·24	3	7·5	7·7	·514	B.
Ho. 541 ...	14 16	77 25	96·6	2·16	2	9·3	10·2	·388	W.B.
Σ 1834 ...	14 17	41 5	343·2	0·14	2	7·1	7·2	·319	B.
Ho. 542 ..	14 23	68 59	264·9	0·58	2	8·8	8·8	·301	B.
Σ 1865 ...	14 37	75 53	145·9	0·43	4	3·5	3·9	·339	B.
OΣ 285 ...	14 42	47 14	110·6	0·32	4	7·1	7·6	·463	B.
OΣ 287 ...	14 48	44 42	321·7	0·79	4	7·5	7·6	·463	B.
Ho. 389 ...	14 48	69 20	98·3	1·46	1	7·0	9·3	·427	W.B.
OΣ 288 ...	14 49	73 56	188·0	1·79	2	6·4	7·1	·340	W.B.
			185·9	1·61	1	...		·400	L.
Σ 1932 ...	15 14	62 50	164·5	0·72	1	5·6	6·1	·400	L.
Σ 1944 ...	15 23	83 36	324·0	1·25	1	7·5	8·1	·427	W.B.
Σ 1954 ...	15 31	79 9	185·5	3·68	2	3·2	4·1	·384	W.B.
OΣ 298 ...	15 33	49 54	191·5	1·11	2	7·0	7·3	·607	B.
Σ 1967 ...	15 39	63 25	113·3	0·63	1	4·0	7·0	·400	L.
OΣ 303 ...	15 57	76 28	145·2	0·84	1	7·4	7·9	·427	W.B.
Σ 2021 ...	16 9	76 14	336·6	3·84	2	6·7	6·9	·384	W.B
Σ 2026 ...	16 12	82 24	187·9	0·50	1	8·6	9·1	·496	L.
			189·4	0·49	1	...		·498	W.B.
OΣ 309 ...	16 16	48 7	64·3	0·42	3	7·5	7·8	·502	B.
Σ 2049 ...	16 24	63 49	204·0	1·17	1	6·5	7·5	·400	L.
Hu. 483 ...	16 24	68 54	295·0	1·59	2	8·2	13·0	·529	W.B.
Σ 2052 ...	16 25	71 24	271·0	1·14	1	7·5	7·5	496	L.
			269·8	1·27	2	...		·505	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910.	N.P.D. 1910.	Position Angle.	Distance.	No. of Nights	Mags.		Epoch 1907.	Obs.
	h m	° ′	°	″					
Σ 2055 ...	16 26	87 49	62·4	1·27	7	4·0	6·1	·524	W.B.
Hu. 487 ...	16 38	67 58	28·9	0·51	2	9·0	9·0	·559	B.
Σ 2084 ...	16 38	58 14	169·3	1·04	4	3·0	6·5	·554	B.
			163·4	1·08	2	...		·419	L.
			168·5	1·31	3	...		·558	W.B.
Σ 2091 ...	16 39	48 38	120·9	0·89	2	7·5	8·0	·604	W.B.
OΣ 315 ...	16 46	88 37	152·1	0·65	3	6·2	8·1	·544	W.B.
Σ 2106 ...	16 47	80 26	298·4	0·19	1	6·7	8·4	·578	B.
Ait. 1142	16 53	75 8	305·5	1·69	1	8·7	12·7	·630	L.
Σ 2114 ...	16 58	81 25	160·7	1·18	3	6·2	7·4	·495	W.B.
Hu. 1176 ...	17 5	53 57	111·7	0·15	3	6·0	6·0	·686	B.
Σ 2132 ...	17 7	93 56	112·0	1·43	1	8·3	9·0	·545	H.F.
Hu. 170 ...	17 8	80 8	278·4	1·88	1	8·5	10·8	·575	W.B.
Ho. 414 ...	17 19	63 50	88·5	0·60	2	8·4	8·8	·723	W.B.
Ho. 415 ...	17 19	64 10	332·9	0·96	2	8·0	8·7	·723	W.B.
Hu. 1179 ...	17 21	51 20	274·5	0·22	4	7·0	7·1	·655	B.
Σ 2171 ...	17 24	99 55	67·9	1·69	3	7·5	7·6	·559	B.
Σ 2173 ...	17 25	90 59	312·3	0·49	3	6·0	6·3	·559	B.
Ho. 417 ...	17 26	51 59	152·4	0·38	3	8·0	8·0	·635	B.
Hu. 179 ...	17 27	78 43	55·4	2·01	1	8·8	8·9	·471	W.B.
Ho. 420 ...	17 35	52 58	107·9	1·48	2	9·3	9·6	·626	B.
Σ 2205 ...	17 40	72 14	306·3	2·01	4	8·0	8·7	·501	W.B.
Σ 2215 ...	17 41	72 15	285 7	0·81	3	5·9	7·9	·511	W.B.
Σ 2220 BC	17 42	62 13	80·1	1·14	2	9·5	10·5	·526	H.F.
= AC 7			79·6	1·28	1	...		·630	L.
OΣ 338 ...	17 48	74 39	14·0	0·76	1	6·6	6·9	·712	B.
Ho. 442 ...	17 48	95 18	94·7	3·60	3	8·2	9·0	·566	W.B.
Ait. 235 ...	17 49	65 0	78·5	0·33	1	7·9	8·1	·712	B.
β 1299 ...	17 52	79 0	154·2	0·42	1	8·5	8·5	·712	B.
Σ 2262 ...	17 58	98 11	259·8	1·78	2	5·0	5·7	·526	H.F.
OΣ 534 ...	18 1	68 34	273·6	2·11	2	7·5	9·5	·549	B.
Σ 2272 ...	18 1	87 28	164·5	2·62	16	4·5	6·0	·595	W.B.
OΣ 341 ...	18 2	68 34	92·0	0·37	3	6·4	7·7	·565	B.
OΣ 524 ...	18 4	70 21	56·7	0·39	3	7·0	8·3	·600	W.B.
Σ 2283 ...	18 5	83 52	82·2	0·90	4	7·2	7·7	·568	B.
Hu. 314 ...	18 5	71 23	152·1	0·56	3	8·3	8·5	·600	W.B.
Hu. 315 ...	18 5	66 27	46·8	0·43	3	9·3	9·3	·675	W.B.
Σ 2289 ...	18 6	73 32	229·3	1·15	4	6·0	7·1	·518	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle.	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Σ 2294 ...	18 9	89 51	281·3	0·53	2	7·4	7·7	·526	H.F
Ho. 430 ...	18 18	69 32	194·7	2·93	1	8·5	9·0	·687	L.
Lewis 20 ...	18 18	69 30	332·1	2·32	1	9·0	9·5	·687	L.
* ...	18 18	70 2	138·0	3·77	1	10·5	11·0	·687	L.
Hu. 581 ...	18 19	75 5	130·2	0·30	2	8·4	9·0	·654	B.
			122·5	0·38	3	...		·706	W.B.
Ho. 83 ...	18 20	62 32	268·2	0·61	2	8·8	8·8	·583	W.B.
... ...			75·3	0·56	1	...		·687	L.
Σ 2320 ...	18 24	65 25	4·7	1·54	3	7·1	9·0	·495	W.B.
OΣ 354 ...	18 28	83 17	170·6	0·82	3	7·2	8·0	·580	W.B.
OΣ 358 ...	18 32	73 6	191·2	1·86	3	6·8	7·2	·533	W.B.
Hu. 198 ...	18 34	81 15	189·1	0·15	1	8·5	8·6	·657	W.B.
Ho. 437 AB	18 37	58 27	120·2	0·43	3	8·3	8·5	·739	W.B.
... CD			339·4	3·55	2	11·2	11·7	·731	W.B.
Ho. 88 ...	18 41	99 38	211·1	2·31	1	9·0	9·0	·578	B.
Σ 2375 ...	18 41	84 36	112·5	2·32	3	6·2	6·6	·517	W.B.
Σ 2422 ...	18 54	64 2	92·7	0·67	2	7·6	7·7	·529	W.B.
β 648 ...	18 54	57 13	179·0	0·42	1	6·0	9·5	·687	L.
			182·2	0·51	2	...		·731	W.B.
Hu. 330 ...	18 55	70 31	28·4	0·77	2	9·0	9·3	·742	W.B.
* ...	18 55	70 32	293·0	2·42	1	9·5	10·5	·668	L.
Hu. 678 ...	18 56	77 54	360·2	2·98	2	8·0	10·5	·507	W.B.
Ho. 92 ...	18 57	57 37	39·5	1·12	2	9·0	9·1	·762	W.B.
Σ 2437 ...	18 58	70 58	58·5	0·80	2	7·8	8·0	·507	W.B.
Ho. 93 ...	18 58	75 42	329·3	1·13	4	7·7	12·0	·711	W.B.
Σ 2455 ...	19 3	67 58	68·8	4·00	1	7·2	8·3	·668	L.
			72·4	3·73	3	...		·698	W.B.
* ...	19 3	67 8	296·2	3·73	1	9·5	10·0	·668	L.
			291·7	4·03	1	...		·715	W.B.
Ho. 444 ...	19 5	63 14	76·7	1·36	3	8·4	10·0	·620	W.B.
Ho. 98 ...	19 5	63 3	146·2	0·31	4	8·0	8·0	·719	W.B.
Ho. 442 ...	19 5	70 55	94·7	3·60	3	9·0	10·5	·566	W.B.
* ...	19 5	62 27	87·5	2·95	1	9·0	9·5	·668	L.
* ...	19 5	62 27	78·5	3·08	1	9·0	10·0	·668	L.
Ho. 447 ...	19 13	62 14	174·4	1·93	3	9·5	9·5	·593	W.B.
Ho. 103 ...	19 14	93 37	246·6	2·45	1	9·2	9·2	·578	B.
Ho. 105 ...	19 18	73 31	187·5	2·65	3	8·5	10·0	·517	W.B.
Hu. 339 ...	19 21	71 32	45 9	0·59	5	8·6	8·6	·596	W.B.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 450 AB	19 23	51 22	267·6	0·97	1	8·0	8·7	·928	H.F.
Σ 2525 ...	19 23	64 52	305·2	0·67	5	7·5	7·7	·558	W.B.
Hu. 340 ...	19 24	71 43	119·9	0·87	4	9·0	9·3	·609	W.B.
Hu. 949 ...	19 30	57 5	103·0	0·69	1	8·6	9·0	·928	H.F.
Ho. 108 ...	19 30	56 43	47·9	0·54	1	8·0	8·0	·928	H.F.
Ho. 110 ...	19 35	71 31	80·5	2·67	3	9·5	9·5	·679	W.B.
Hu. 953 ...	19 36	54 58	177·0	1·08	2	8·6	9·0	·762	W.B.
Ho. 112 ...	19 37	71 35	70·2	2·72	3	9·0	9·0	·675	W.B.
Σ 2576 ..	19 42	56 38	287·9	2·45	2	7·8	7·8	·762	W.B.
AGC 11 ...	19 45	71 5	140·4	0·19	3	5·5	6·5	·705	B.
Σ 2596 ...	19 50	74 59	322·0	2·02	3	7·2	8·6	·692	W.B.
* ...	19 50	67 47	81·8	3·12	1	10·5	10·5	·657	W.B.
Σ 2600 ...	19 51	67 47	56·4	3·09	3	8·3	9·7	·673	W.B.
BD+21° 3994 AB	19 54	68 6	275·7	1·32	3	9·0	10·4	·713	W.B.
			277·8	1·33	1	...		·928	H.F.
BD+21°3996	19 55	68 11	298·9	1·94	3	9·0	9·5	·694	W.B.
Σ 2607 AB =OΣ 392	19 55	48 1	292·1	0·65	1	7·2	9·0	·928	H.F.
Ho. 583 ...	19 55	68 8	256·0	1·21	1	9·0	10·7	·767	W.B.
Ho. 118 ...	19 59	56 35	22·7	2·74	2	9·1	11·0	·762	W.B.
Ait. 1194 ...	20 0	77 54	305·1	0·62	1	8·9	9·0	·750	B.
			299·6	0·80	1	...		·767	W.B.
Ho. 122 ...	20 11	61 37	75·4	1·10	1	9·0	9·7	·756	W.B.
Ho. 591 ...	20 14	62 23	294·0	2·30	2	9·5	10·0	·655	W.B.
* ...	20 16	51 16	198·6	1·95	1	7·0	11·3	·928	H.F.
Ho. 126 ...	20 16	51 18	152·3	2·86	1	9·7	9·7	·928	H.F.
Hu. 1198 ...	20 20	77 17	32·8	0·65	1	8·4	9·2	·835	H.F.
Ho. 457 ...	20 21	60 55	60·0	2·13	3	8·1	8·1	·692	W.B.
Σ 2690 BC	20 27	79 5	217·2	0·41	2	7·5	7·5	·731	B.
Ho. 133 ...	20 28	103 51	184·3	0·87	1	8·0	8·0	·750	B.
Ho. 134 ...	20 32	55 9	203·2	3·36	2	8·5	8·7	·762	W.B.
β 151 ...	20 33	75 45	125·5	0·25	4	4·7	6·1	·730	B.
Ho. 458 ...	20 35	61 10	279·0	2·12	3	9·0	9·1	·673	W.B.
Σ 2716 ...	20 37	58 3	49·7	2·65	1	6·0	8·2	·835	H.F.
Hu. 690 ...	20 41	56 11	290·1	0·93	1	9·0	9·2	·928	H.F.
Σ 2723 ...	20 42	74 28	103·4	1·37	3	7 3	8·0	·692	W.B.
Ho. 141 ...	20 43	71 2	291·0	1·16	1	8·4	10·8	·619	W.B.

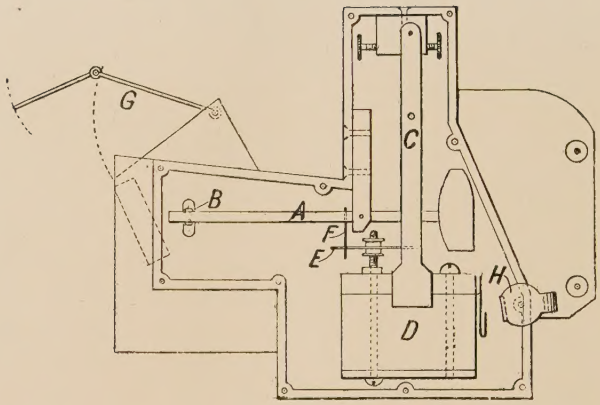
Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 144 ...	20 48	70 13	164·9	0·43	1	7·0	7·0	·657	W.B.
Ho. 146 ...	20 51	55 7	51·9	0·56	1	8·0	8·1	·928	H.F.
Hu. 764 ...	20 56	55 55	190·2	0·48	1	7·5	8·7	·928	H.F.
Ho. 282 ...	21 0	44 30	359·0	0·28	1	7·0	7·0	·805	B.
Hu. 767 ...	21 9	74 24	143·7	0·18	2	7·0	7·0	·738	B.
OΣ 535 ...	21 10	80 22	171·9	0·16	1	4·5	5·0	·687	L.
			158·2	[0·15]	3	...		·706	W.B.†
			95·0	0·13	3	...		·718	B.
			76·5	0·14	3	...		·848	B.
			136·0	0·11	3	...		·862	W.B.
Ho. 284 ...	21 12	74 24	90·2	3·73	2	9·0	9·3	·636	W.B.
Ho. 153 ...	21 14	56 37	121·2	0·78	1	8·0	9·0	·928	H.F.
Ho. 286 ...	21 16	52 8	304·9	0·16	1	6·0	6·0	·764	B.
Ho. 155 ...	21 16	57 8	31·7	2·02	2	8·0	9·5	·783	W.B
Ho. 157 ...	21 19	58 22	23·1	3·69	2	7·7	7·7	·783	W.B.
Ait. 1219 ...	21 21	60 8	120·8	1·40	2	9·3	9·4	·762	W.B.
Ait. 1220 ...	21 21	58 54	145·5	1·73	2	8·5	9·0	·762	W.B.
Ho. 160 ...	21 25	47 29	178·9	1·72	1	8·3	9·0	·928	H.F.
Hu. 371 ...	21 31	65 57	162·1	0·22	3	7·0	7·5	·742	B.
Hu. 372 ...	21 34	66 48	149·6	0·30	3	9·0	9·0	·742	B.
Hu. 280 ...	21 38	84 30	134·6	0·18	3	7·7	8·1	·742	B.
Hu. 374 ...	21 39	66 32	32·8	0·33	3	9·0	9·0	·742	B.
Ho. 605 ...	21 40	55 32	344·0	2·09	1	9·2	9·9	·942	L.
β 989 ...	21 41	64 46	95·1	0·19	9	4·8	5·3	·806	W.B.
			93·1	0·08	1	...		·687	L.
			97·6	0·13	5	...		·735	B.
Ho. 166 ...	21 46	62 34	75·3	0·34	2	7·5	7·5	·782	W B.
Ho. 169 ...	21 46	54 19	137·1	3·18	1	8·0	12·0	·942	L.
Ho. 467 ...	21 46	68 10	182·6	1·15	2	8·0	10·2	·712	W.B.
Ho. 179 ...	22 4	60 14	261·5	0·42	1	8·0	9·0	·810	W.B.
Ho. 182 ...	22 19	72 54	128·9	1·46	3	8·5	8·5	·719	W.B.
Σ 2900 ...	22 19	69 39	175·8	1·41	1	6·0	9·2	·942	L.
Ho. 475 ...	22 29	64 3	315·9	0·74	2	8·0	8·2	·769	W.B.
Ho. 294 ...	22 33	63 2	58·1	2·03	2	8·0	10·0	·763	W.B.
Ho. 480 ...	22 34	60 45	228·7	0·69	1	8·0	9·1	·810	W.B.

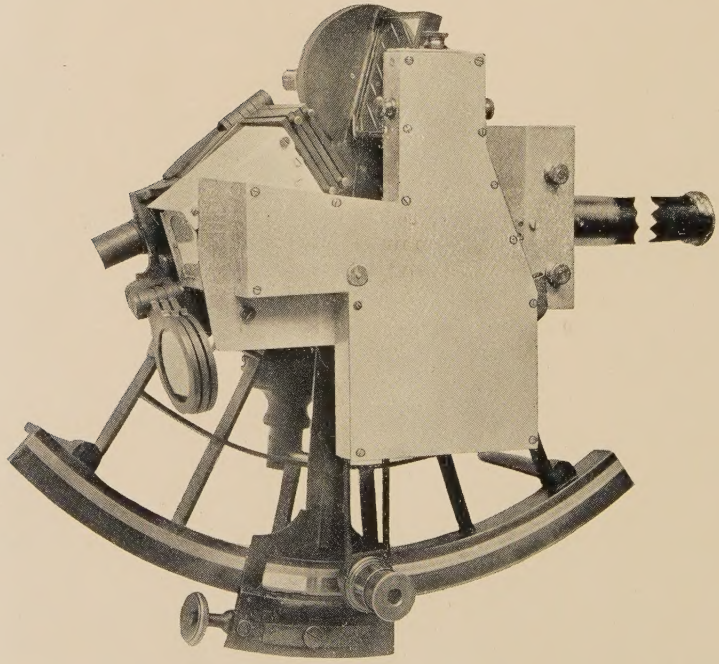
† Distance estimated.

Micrometer Measures of Double Stars—continued.

Star's Name.	R.A. 1910. h m	N.P.D. 1910. ° ′	Position Angle. °	Distance. ″	No. of Nights	Mags.		Epoch 1907.	Obs.
Ho. 296 ...	22 36	75 56	61·7	0·46	3	5·5	5·5	·742	B.
			64·0	0·33	2	...		·791	W.B.
Hu. 780 ...	22 37	75 56	13·9	0·33	3	9·2	9·4	·742	B.
			5·3	0·51	1	...		·942	L.
Σ 2934 ...	22 37	69 5	134·3	0·63	1	8·2	9·2	·942	L.
Hu. 781 ...	22 38	75 16	311·9	0·30	1	8·5	9·5	·862	W.B.
Ho. 481 ...	22 41	61 6	114·1	0·31	3	8·3	8·3	·742	B.
Ho. 190 ...	22 45	62 28	153·9	1·86	1	9·2	9·2	·810	W.B.
Ho. 482 ...	22 47	64 5	56·5	0·22	5	6·8	6·8	·772	B.
Hu. 987 ...	22 51	74 42	243·8	0·54	3	8·6	8·8	·780	B.
			252·4	0·93	1	...		·942	L.
Ho. 192 ...	22 55	60 24	30·4	1·47	2	8·5	9·5	·763	W.B.
Ho. 193 ...	22 59	60 35	165·7	2·42	1	7·2	11·2	·871	W.B.
Ho. 196 ...	23 4	60 1	288·5	1·94	1	8·0	11·0	·871	W.B.
Ho. 198 ...	23 13	73 38	32·8	2·31	3	8·0	9·0	·860	W.B.
Hu. 400 ...	23 13	72 11	229·4	0·33	3	7·4	8·8	·780	B.
Ho. 489 ...	23 22	62 47	245·0	0·45	2	7·8	7·8	·832	W.B.
Σ 3012 ...	23 23	73 55	189·9	2·99	1	8·7	8·8	·942	L.
Σ 3013 ...	23 23	73 55	273·8	3·62	1	7·8	9·3	·942	L.
Hu. 298 ...	23 28	83 25	76·1	0·28	2	6·8	7·4	·757	B.
*	23 36	70 8	128·2	1·60	1	8·0	11·0	·942	L.
Hu. 300 ...	23 41	84 1	119·0	1·14	3	8·7	9·0	·780	B.
			118·8	1·15	1	...		·893	W.B.
Ho. 206 ...	23 55	56 14	189·0	1·69	1	8·0	10·0	·810	W.B.



Details of the Pendulum attachment.



Sextant with Artificial horizon attachment.

Note on "the Spanner," Captain Gadsden's Artificial Horizon for Sextants. By H. H. Turner, D.Sc., F.R.S., Savilian Professor. (Plate 13.)

It seems desirable that the existence of an instrument which has been found to work in practice, and which is useful to a sailor in fog or at night, should be made widely known. I venture to emphasise the fact that the instrument has been found practically useful; for this point is essential. There have been many promising suggestions for such an instrument, which have failed on actual trial.

The apparatus is attached to an ordinary sextant, and is the invention of Captain H. A. Gadsden, of 6 Binden Road, Ravenscourt Park W., London, to whom I must refer those interested for further details. The subjoined figures (Plate 13) will explain its general nature. The instrument was exhibited to the Society at the April meeting.

An artificial horizon is provided, which consists of two bars that exactly span the periphery of the Sun's image in the horizon glass, and are maintained in horizontal position by means of a pendulum forming part of the attachment. This is shown in detail in Plate 13. At *A* is a pivoted balance beam which carries the horizon bars *B*, the latter extending through a slot in the casing. The pendulum comprises a pair of plates *C*, pivoted to a block at their upper ends, while their lower ends carry a heavy weight *D*. A plate *E* is secured to the weight *D*, and is formed with an opening through which extends a rod *F*, projecting from the beam *A*. The pendulum maintains its vertical position when the sextant is held in the hand, and by means of the rod *F* the horizon bars are kept at a proper horizontal level.

As the horizon bars are near the eye of an observer, the telescope of a sextant cannot be used. Observations are therefore made with a simple tube, longer than that usually supplied, so that the eye can be properly focussed on the bars.

If it is desired to use the sextant in the ordinary way, the spanner can be "locked" out of sight.

Captain Gadsden has also added to the sextant a stock of simple form (resembling a gunstock), which is put against the shoulder and helps to steady the instrument.

Note on Father Stein's paper "On Dr. Roberts' method of determining the absolute dimensions of an Algol Variable Star."
By Alex. W. Roberts, D.Sc.

Professor Turner has kindly sent me in advance the main conclusions Father Stein has arrived at in his paper in *M.N.* (vol. lxviii. p. 490).

Through unaccountable error, I regarded the value of the light ratio as a maximum when the difference between the masses of the component stars was a minimum. It then, however, is zero.

Although in the case where the masses of the component stars are equal it is impossible to arrive at the absolute magnitude of a binary system, this is not the case where there is disparity between the masses, and I still have the hope that through careful determination of the quadrant points of a close binary orbit we may advance our knowledge in this direction.

[*Note by H. H. Turner.*—It should perhaps be made clear that the above note was written before Dr. Roberts had actually seen Father Stein's paper. On referring to his notes, he found at once the unfortunate slip above stated, and hastened to admit the correction without awaiting the paper itself. His promptness makes it possible to include this note in the present number of the *Notices.*]